COMPUTER ENGINEERING

DISCRETE MATHEMATICS

(CBCGS - DEC 2017 SEM 3)

Q1.a) Prove that 1.1! + 2.2! + 3.3! + … + n.n! = (n+1)! -1 where n is a positive integer. [5]

Solution:-

Let $p(n)$ = 1.1! + 2.2! + 3.3! + -- + n.n!

Let us prove $p(1)$ is true;

$n = 1$; => LHS : 1.1! = 1

RHS : (n+1)! = -1 => (2)! – 1 => 2-1 =1

Let us assume $p(k)$ is true

1.1! + 2.2! + 3.3! + … + k.k! = (k+1)! -1 ……………………… (1)

LHS :- 1.1! + 2.2! + 3.3! + … + k.k! + (k+1)(k+1)!

=> (k+1)! – 1 + (k+1)(k+1)!

=> (k+1)! + (k+1)(k+1)! - 1

=> (k+1)![1+k+1] – 1

=> (k+1)![k+2] – 1

=> (k+2)! – 1

= RHS

Thus the result is proved.

Q1.b) Let A={a,b,c}. Show that {P(A), ⊆ } is a poset and draw its Hasse diagram. [5]

Solution:-

Set contained belongs is always a partial order since for any subset B of A; B is a subset of B is reflexive.

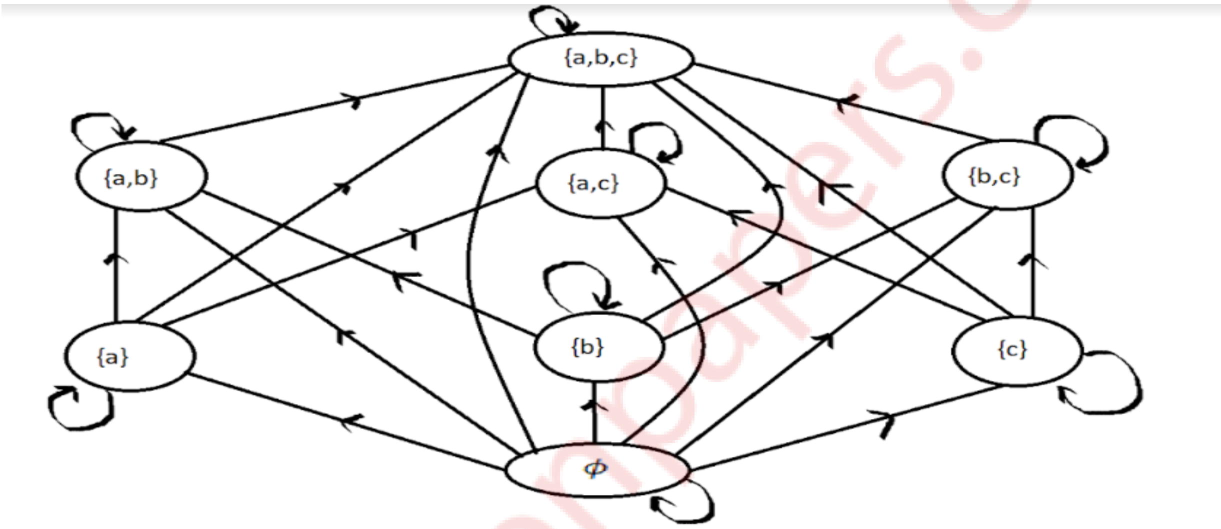
If B ⊆ C and C ⊆ => B = C ; so ⊆ is anti symmetric

If B ⊆ C, C ⊆ D then B ⊆ D . So ⊆ is transitive

Partial order relation of set containment on set P(A) is as follows:-

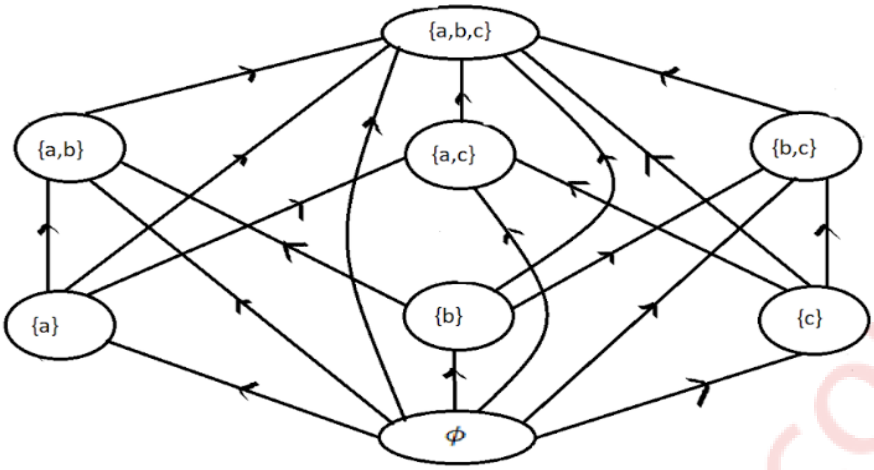
R = {{φ},{φ}}; {{φ},{a}} ; {{φ},{b}}; {{φ},{c}}; {{φ},{a,b}}; {{φ},{a,c}}; {{φ},{b,c}}
{{φ},{a,b,c}};{{a},{a}}; {{a},{a,b}}; {{a},{a,c}}; {{a},{a,b,c}};{{b},{b}};{{b},{a,b}}
{{b},{b,c}};{{b},{a,b,c}};{{c},{c}};{{c},{a,c}}; {{c},{b,c}};{{c},{a,b,c}};{{a,b},{a,b,c}}
{{b,c},{a,b,c}};{{a,b},{a,b}};{{a,c},{a,c}};{{b,c},{b,c}};{{a,b,c},{a,b,c}}

Diagraph:-



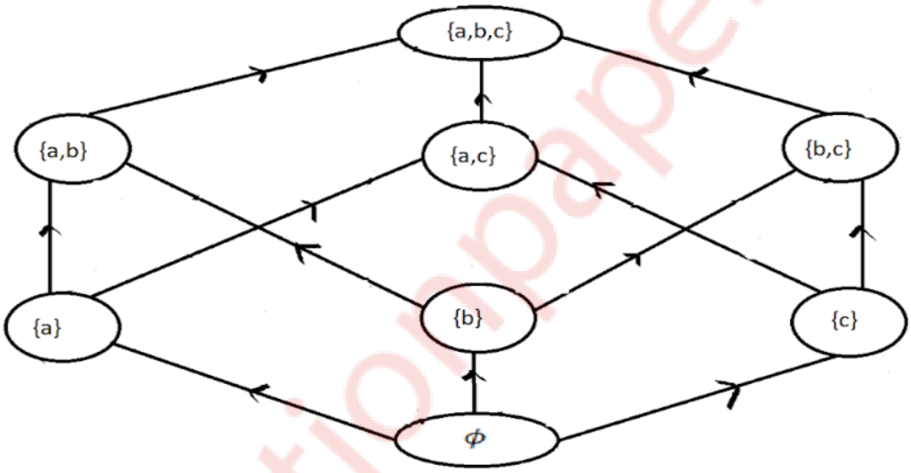
	{φ}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}	{a,b,c}
{φ}	1	1	1	1	1	1	1	1
{a}	0	1	0	0	1	1	0	1
{b}	0	0	1	0	1	0	1	1
{c}	0	0	0	1	0	1	1	1
{a,b}	0	0	0	0	1	0	0	1
{a,c}	0	0	0	0	0	1	0	1
{b,c}	0	0	0	0	0	0	1	1
{a,b,c}	0	0	0	0	0	0	0	1

Step 1:- remove loops:



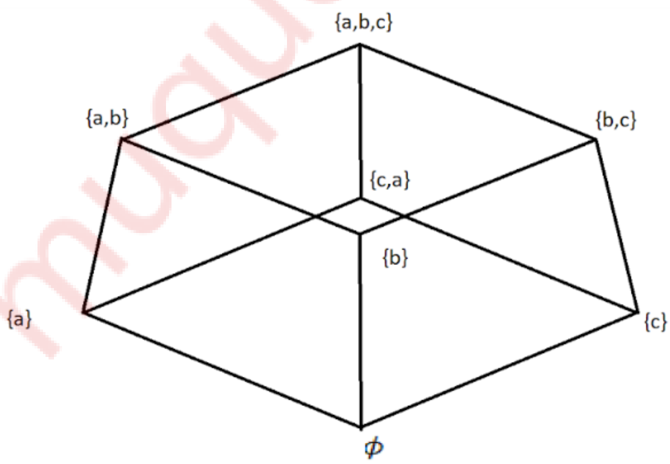
Remove transitive edges:-

{{φ},{a,b}},{{φ},{a,c}},{{φ},{b,c}},{{φ},{a,b,c}},{{a},{a,b,c}},{{b},{a,b,c}},{{c},{a,b,c}}



All edges are pointing upwards. Now replace circles by dots and remove arrows from edges.

Hasse Diagram:-



Q1.c) Explain the following terms:

[5]

- **i. Lattice**
- **ii. Poset**
- **iii. Normal Subgroup**
- **iv. Group**
- **v. Planar Graph**

Solution:-

1. **<u>Lattice</u>** :- It is a poset $(L, \leq)$ in which every subset {a,b} consisting of two elements has a least upper bound and a greatest lower bound. We denote LUB({a,b}) by $a \vee b$ and call it the join of a,b. similarly we denote GLB ({a,b}) by $a \wedge b$ and call it the meet of a and b.

2. **<u>Poset:-</u>** A relation R on a set A is called partial order if R is reflexive, anti symmetric and transitive poset. The set A together with the partial order R is called a partial order set or simply a poset.

3. **<u>Normal Subgroup :-</u>** A subgroup μ of G is said to be a normal subgroup of G if for every $a \in G$, aH = Ha. A subgroup of an Abelian group is normal.

4. **<u>Group :</u>**- Let (A,*) be an algebraic system where * is a group if the following conditions are satisfied.

   1. * is closed operation.
   2. * is an associative operation
   3. There is an identity operation.
   4. Every element in A has a left inverse.

Because of associativity ,a left inverse of an element is also a right inverse of the element in a group.

5. **Planar Graph:-** A graphic is said to be planar if it can be drawn on a plane in such a way that no edges cross one another except of course at common vertices.

Q1.d) Comment whether the function f is one to one or onto. Consider the function [5]

Solution:-

f:N⟶N where N is set of natural numbers including zero.

$f(j) = j^2 + 2$

Solution:-

$f(j) = j^2 + 2$

N = {0,1,2,3 ……….}

$f(0) = 2$

$f(1) = 3$

$f(2) = 6$

$f(3) = 11$

$f(4) = 18$

For every number n ∈ N we can find another n. so the given function is one to one.

But not every element of n ∈ N is image of some element n. so the given function is not onto.

Q2.a) Find the number of ways a person can be distributed Rs 601 as pocket money to his three sons, so that no son should receive more than the combined total of the other two. (Assume no fraction of a rupee is allowed)
Solution:- [6]

Let A,B and C be the 3 sons and a,b and c be the money given to them respectively. By the given conditions;0

$a \leq b + c$, $a + b + c = 601c=601$

-a

$a = 300$

Similarly, we can deduce that

$b \le 30000$ and $c \le 300$

So we have,

$a \le 300$, $b \le 300$, $c \le 300300$ and $a + b + c = 601$

The corresponding multinomial is

$(1+x+x^2+x^3+ \ldots\ldots + x^{300})^3$

The total number of distribution is the coefficient of x^{601} in the expansion of $(1+x+x^2+x^3+ \ldots\ldots + x^{300})^3$

$$(1+x+x^2+x^3+ \ldots\ldots + x^{300})^3 = \left(\frac{x^{301}-1}{x-1}\right)^3 = -(x^{301}-1)^3(1-x)^{-3}$$

$$(1+x+x^2+x^3+ \ldots\ldots + x^{300})^3 = -(x^{903} - 3x^{602} + 3x^{301} - 1)\times\left(1+\binom{3}{1}x+\binom{4}{2}x^2+\ldots\ldots+\binom{603}{601}x^{601}\right)$$

Hence the coefficient of x^{601} in the above expression is

$$\binom{603}{601} - 3\times\binom{302}{300} = 45150$$

Q2.b) Let A = {a_1,a_2,a_3,a_4,a_5} and let R be a relation on A whose matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find M_R * by Warshall's algorithm. **[6]**

Solution:-

A = {a_1 , a_2, a_3 , a_4, a_5 }

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$M_R = W_0$

$$W_0 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Computing W_1 :

C_1 = 1 is present at 1,4

R_1 = 1 is present at 1,4

Put 1 in (1,1);(1,4);(4,1);(4,4)

$$W_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Computing W_1

C_2 = 1 is present at 2,5

R_2 = 1 is present at 2

Put 1 in (2,2) and (5,2)

$$W_2 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Computing W_3

C_3 = 1 is present at no position

R_3 = 1 is present at 4,5

No new ordered pair , therefore $W_3 = W_2$

$$W_3 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Computing W_4

C_4 = 1 is present at 1,3,4

R_4 = 1 is present at 1,4

Put 1 in (1,1);(1,4);(3,1);(3,4);(4,1);(4,4)

$$W_4 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Computing W_5

C_5 = 1 is present at 3,5

R_5 = 1 is present at 2,5

Put 1 in (3,2);(3,5);(5,2);(5,5)

$$W_5 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$M_R^{\infty} = W_5$

Transitive closure = $\{(a_1,a_1),(a_1,a_4),(a_2,a_2),(a_3,a_1),(a_3,a_2),(a_3,a_4),(a_3,a_5),(a_4,a_4),(a_5,a_2),(a_5,a_5)\}$

Q2.c) Find the complete solution of the recurrence relation : [4]

$a_n + 2a_{n-1} = n + 3$ for n≥1 and with $a_0 = 3$

Solution:-

$a_0 = 3$

Recurrence relation :- $a_n + 2a_{n-1} = n + 3$

The characteristics equation is:-

$\alpha + 2 = 0$

$\alpha = -2$

Homogenous solution is :

$a_n^{(n)} = A_1(-2)^n$

For particular solution:

RHS is linear polynomial, the particular solution will be of the form $P_0 + P_1^n$

$a_n = p_0 + p_1 n$

$a_{n-1} = p_0 + p_1(n-1)$

Substituting these values in given equation

$P_0 + P_1 n + 2[P_0 + P_1(n-1)] = n + 3$

$P_0 + P_1 n + 2P_0 + 2P_1 n - 2P_1 = n + 3$

$(3P_0 - 2P_1) + 3P_1 n = n + 3$

Comparing coefficient on both sides

$3P_0 - 2P_1 = 3$; $3P_1 = 1$

$P_1 = \frac{1}{3}$

$3P_0 - \frac{2}{3} = 3$

$3P_0 = \frac{11}{3}$

$P_0 = \frac{11}{9}$

Thus the general solution is:

$a_n = a_n^{(n)} + a_n^{(p)}$

$a_n = A_1(-2)^n + \frac{11}{9} + \frac{1}{3}$

Using initial condition ; $a_0 = 3$

$$a_n = 1.78(-2)^n + \frac{11}{9} + \frac{1}{3}n$$

Q2.d) Let f:R→R defined as $f(x) = x^3$ and g:R→R defined as $g(x) = 4x^2 + 1$
Find out gof,fog,f^2,g^2 **[4]**

Solution:-

1. $gof = g(f(x)) = g(x^2) = 4(x^3)^2 + 1 = 4x^6 + 1$

2. $fog = f(g(x)) = f(4x^2 + 1) = (4x^2+1)^3$

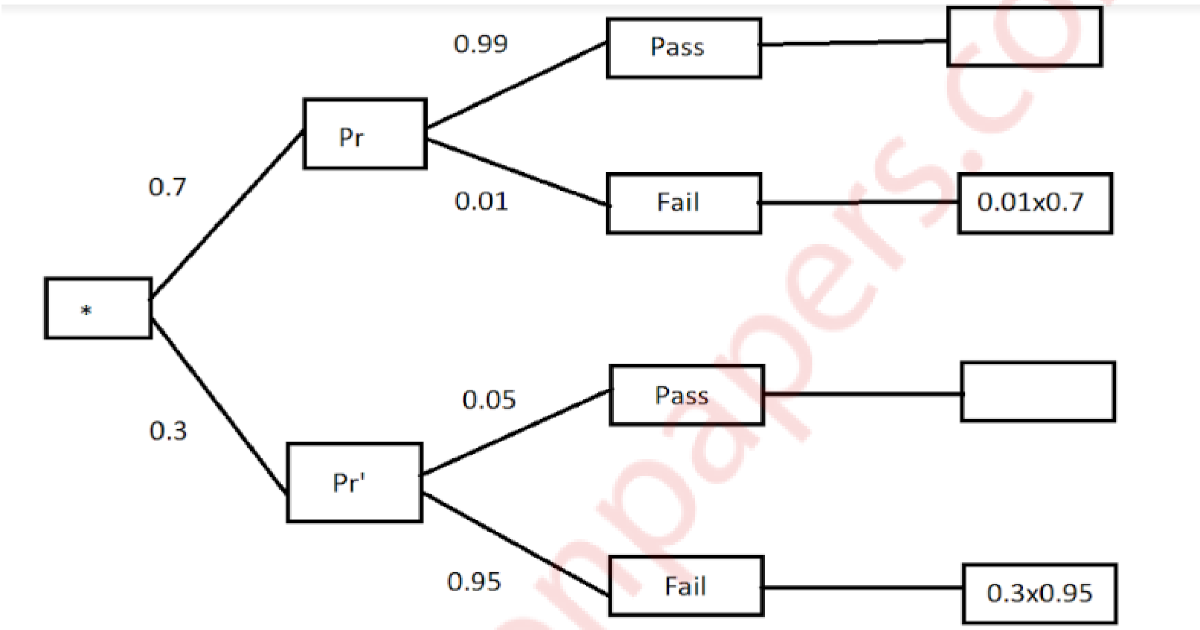
3. $f^2 = fof = f(f(x)) = f(x^3) = (x^3)^3$

4. $g^2 = gog = g(g(x)) = g(4x^2 + 1) = 4\left(4x^2+1\right)^2 + 1$

Q3.a) Given that a student had prepared, the probability o passing a certain entrance exam is 0.99. Given that a student did not prepare, the probability of the passing the entrance is exam is 0.5. Assume that the probability o preparing is 0.7. The student fails in the exam. What is the probability that he or she did not prepare. [6]

Solution:-

Probability tree diagram



We will find:

$P(\text{Fail} \mid Pr') = P(\text{Fail} \cap Pr')$

$P(\text{Fail} \mid Pr') = P(\text{Fail} \cap Pr') + P(\text{Fail} \cap Pr)$

$$P(\text{Fail} \mid Pr') = \frac{0.3 \times 0.95}{0.3\times0.95+0.7\times0.01} = 0.976$$

Probability = 0.976

Q3. b) Define an equivalence relation with example. Let 'T' be a set of triangles in a plane and define R as the set R = {(a,b)|a,b ∈ T and a is congruent to b} then show that R is an equivalence relation. [6]

Solution:-

A relation R on a set A is called an equivalence relation if it is reflexsive symmetric and transitive.

eg:- let A = IR and R be 'quality of numbers '.

Consider all subsets of a universal set and R be the relation 'equality of sets'.

A is set of triangles and R is 'similarity' of triangles.

Digraph of equivalence relation will have a loop. Edge from b,a if a,b is present and if are from a,b and are from b,c; there should be are from a to c.

Let T be set of triangles in a place.

Since every triangle is congruent to itself; R is reflexive.

If Δa is congruent to Δb, then Δb is congruent to Δa; R is symmetric

If Δa ≅ Δb and Δb ≅ Δc implies Δa ≅ Δc;

R is transitive.

Therefore R is equivalence relation.

Q3.c) Let A=B=R, the set of real numbers. Let f:A→B be given by the formula $f(x) = 2x^3 - 1$ and let g:B→A be given by the formula

$g(x) = \sqrt[3]{\frac{1}{2}y+\frac{1}{2}}$. Show that f is bijection between A and B and g is a bijection between B and A. **[4]**

Solution:-

f is bijection if it is one to one and onto $f(x) = 2x^3 - 1$ to be one to one and onto

If a,b ∈ A

Such that f(a) = f(b)

$2a^3 - 1 = 2b^3 - 1$

$a^3 = b^3$

a = b

f is one to one

now for $y = 2x^3 - 1$

$1 + y = 2x^3$

$\frac{1+y}{2} = x^3$

$x = \sqrt[3]{\frac{y+1}{2}}$

For each y ∈ B;there exist unique x in A

Such that f(x) = y

f is onto

f is bijective

y is bijective if it is one to one and onto

g(a) = g(b)

$\sqrt[3]{\frac{a}{2}+\frac{1}{2}} = \sqrt[3]{\frac{b}{2}+\frac{1}{2}}$

Cubing both sides;

$\frac{a}{2} + \frac{1}{2} = \frac{b}{2} + \frac{1}{2}$

a = b

g is one to one.

Now for x ∈ A; there exists unique y in B such that g(y) = x

g is onto.

So g is bijective.

Q3.d) Let z_n denote the set of the integers {0, 1, 2, …, n-1}. Let O be a binary operation on z_n denote such that a O b = the reminder of ab divided by n.

i) Construct the table for the operation O for n=4

ii) Show that (z_n,O) is a semigroup for any n. [4]

Solution:-

1. Table for the operation * for n =4

$*_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3

2	0	2	0	2
3	0	3	2	1

2. The set z_n is closed under the operation * because for any a,b,∈ z_n

   (a * b) ∈ z_n

   $(a *_4 b) *_4 c = a *_4 (b *_4 c)$

   Let a=1 ; b =2; c = 3

   $(1 *_4 2) *_4 3 = 1 *_4 (2 *_4 3)$

   $2 *_4 3 = 1 *_4 (2)$

   2 = 2

Is associative operation

From above deduction ; (z_n, *) is semigroup for n.

Q4.a) **[6]**

1. **Among 50 students in a class, 26 got an A in the first examination and 21 got an A in the second examination. If 17 students did not get an A in either examination, how many students got an A in both examinations?**
2. **If the number of students who got an A in the first examination is equal to that in the second examination. If the total number of students who got an A in exactly one examination is 40 and if 4 students did not get an A in either examination then determine the number of students who got an A in the first examination only, who got A in the second examination only and who got an A in both the examination.**

Solution:

a) Let T be no of students

   Let F be students who got A in 1^{st} exam

   Let S be students who got A in 2^{nd} exam n(T) = 50; n(F) = 26;   n(S) = 21

no of students who did not get on A in either examination = 17

no of students got an A in at least one examination is 50-17 = 33

no of students got A in both exams is n(F∩S)

33 = n(F) + n(S) – n(F∩S)

N(F∩S) = 47-33 = 14

b) Number of students who got an A in 1st exam equal to that in 2nd exam; n(F) = n(S)

Total no of students who got an A in exactly one examination is 40

N(F) + n(S) - 2n(F∩S) = 40   …………….(1)

4 students did not get an A in at least one examination is 50-4 = 46

From (i);

n(F) + n(S) – 2n(F∩S) = 40

n(F) + n(S) -n F∩S) –n(F∩S) = 40

46 – n(F∩S) = 40

n(F∩S) = 6   …………… (2)

6 students got an A in both examinations

Using equation (i)

n(F) + n(S) – 2n(F∩S) = 40

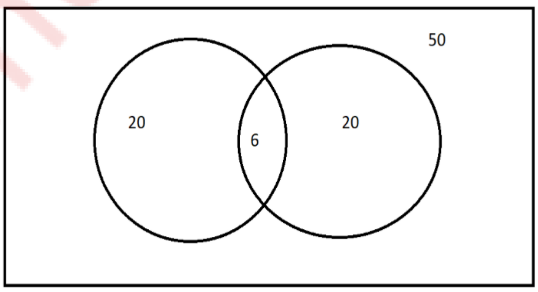
n(F) + n(S) – (2 x 6) =40

n(F) + n(S) = 52

n(F) = n(S) = 52/2 = 26

n(F) – n(F∩S) = 26-6 = 20 got A in first exam

n(S) – n(F∩S) = 26-6 = 20 got A in first exam.



Q4.b) Consider the (2,5) group encoding function **[6]**

e:$B^2 \rightarrow B^5$ defined by :

e(00)=000000 **e(01)=01110**

e(10)=10101 **e(11)=11011**

Decode the following words relative to a maximum likelihood decoding function:

i) 11110 ii) 10011 iii)10100

Solution:-

e : $B^2 \rightarrow B^5$ defined by;

e(00) = 00000 e(01) = 01110

e(10) = 10101 e(11) = 11011

decoding table;

00	01	10	11
00000	01110	10101	11011
00001	01111	10100	11010
00010	01100	10111	11001
00100	01010	10001	11111
01000	00110	11101	10011
10000	11110	00101	01011

1. We receive the word 11110 we first locate it in 2nd column. The word at top is 01110. We decode 11110 as 01
2. We receive the word 10011 we first locate it in 4th column. The word at top is 11011. We decode it as 11
3. We receive the word 10100 we first locate it in 3rd column. The word at top is 10101. We decode it as 10.

Q4.c) (i) Is every Eulerian graph a Hamiltonian? [4]

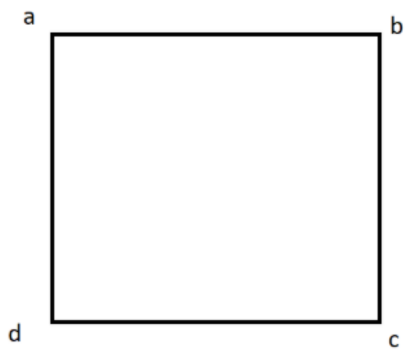
3. Is every Hamiltonian graph a Eulerian?

Explain with the necessary graph.

Solution:-

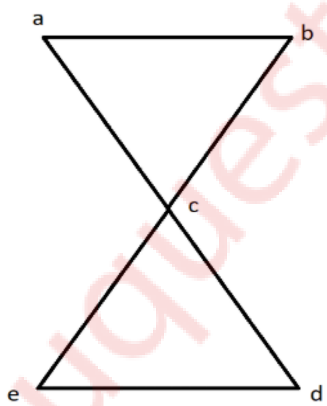
1. Let G = (V,E) be a graph. A eulerian graph is a graph which passes through every edge exactly once.

   Let $G_1(V_1,E_1)$ be a graph. A Hamiltonian circuit is one which passes through every vertex exactly one. A graph is called Hamiltonian if it posses a Hamiltonian circuit.



Eulerian : a,b,c,d,a

Hamiltonian : a,b,c,d,a



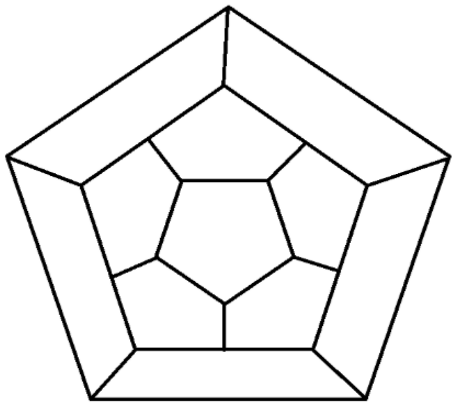
Eulerian : c,a,b,c,e,d,c

No Hamiltonian

Hence every eulerian graph is not Hamiltonian.

2. In Hamiltonian graph, we need to visit each vertex once except last vertex.

Repetition of edge is not necessary. Therefore Hamiltonian graph may not be Eulerian.



Hamiltonian but not eulerian(since it is not possible to cover all edges at once).

Q4.d) Given the parity check matrix. [4]

$$\mathbf{H} = \begin{vmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{vmatrix}$$

Find the minimum distance of the code generated by H. How many errors it can detect and correct?

Solution:-

$$H = \begin{vmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{vmatrix}$$

In the given parity check matrix , all columns are distinct and non zero.

So d >= 3

We can use the property that the minimum distance of a binary linear code is equal to the smallest no of columns of the parity check matrix H that sum upto zero.

We can so sum of first three columns is zero so minimum dist = 3

It can correct $(d_{min} - 1)/2 = 1$ error

It can detect $d_{min} - 1 = 2$ errors.

Q5 a) Explain pigeonhole principle and extended pigeonhole principle. Show that in any room of people who have been doing some handshaking there will always be at least two people who have shaken hands in the same number of times. [6]

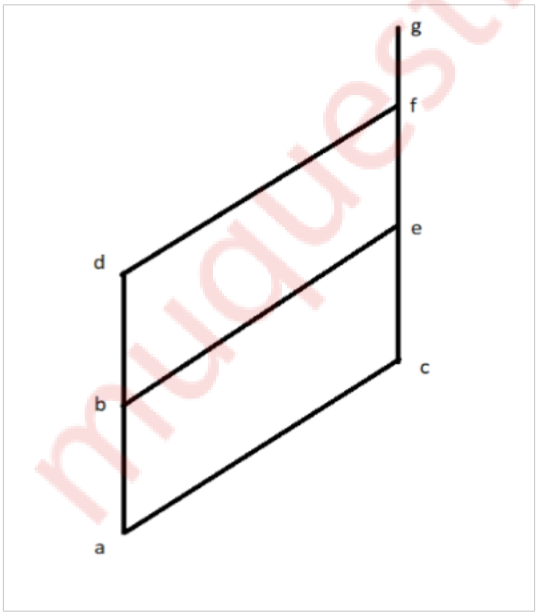
Solution:-

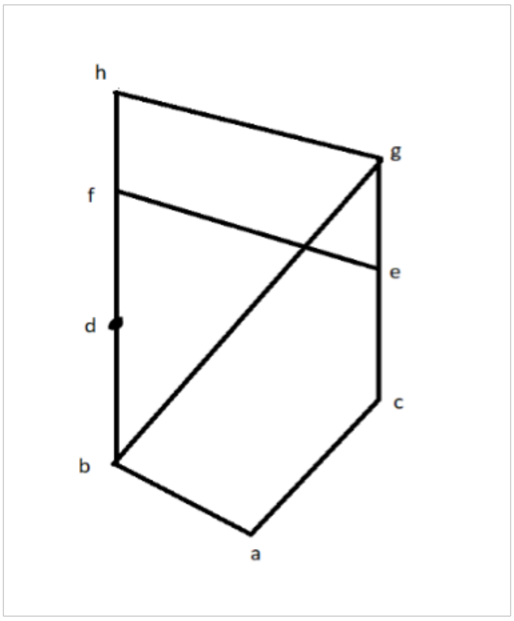
Pigeon hole principle : If n pigeons are assigned to m pigeonholes and m < n then at least one pigeonhole contains two or more pigeons.

Extended pigeonhole : If n pigeons are assigned for m pigeonholes then one of the pigeonholes must obtain at least $\left[\frac{(n-1)}{m}\right] + 1$ pigeons.

There are n people in a party.(n ≥ 2). If no two people have shaken hands with equal number of people then their handshake count must differ by at least 1. So the possible choice for hand shake count would be 0,1 …… n-1. These are exactly n choices and n people. If there exit a person with (n-1) handshake count, there can be a person with 0 handshake count. Thus reducing the possible choices to (n-1). Now due to pigeonhole principle , we have that at least two person will have same number of handshake count.

Q5.b) Determine whether the poset with the following Hasse diagrams are lattice or not. Justify your answer. [6]

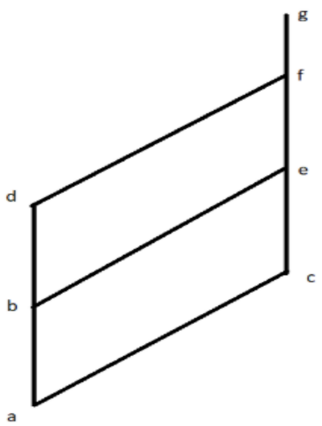




Solution:-

1. LUB :-

V	a	b	c	d	e	f	g
a	a	b	e	d	e	f	g
b	b	b	e	d	e	f	g
c	c	e	c	f	e	f	g
d	d	d	f	d	f	f	g
e	e	e	e	f	e	f	g
f	f	f	f	f	f	f	g
g	g	g	g	g	g	g	g



GLB:-

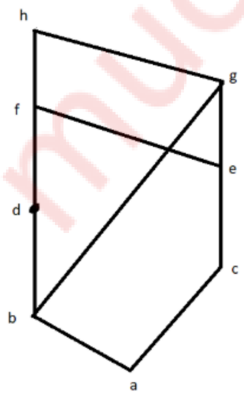
^	a	b	c	d	e	f	g
a	a	a	a	a	a	a	a
b	a	b	a	b	b	b	b
c	a	a	c	a	c	c	c
d	a	b	a	d	b	d	d
e	a	b	c	b	e	e	e
f	a	b	c	d	e	f	f
g	a	b	c	d	e	f	g

2. LUB:-

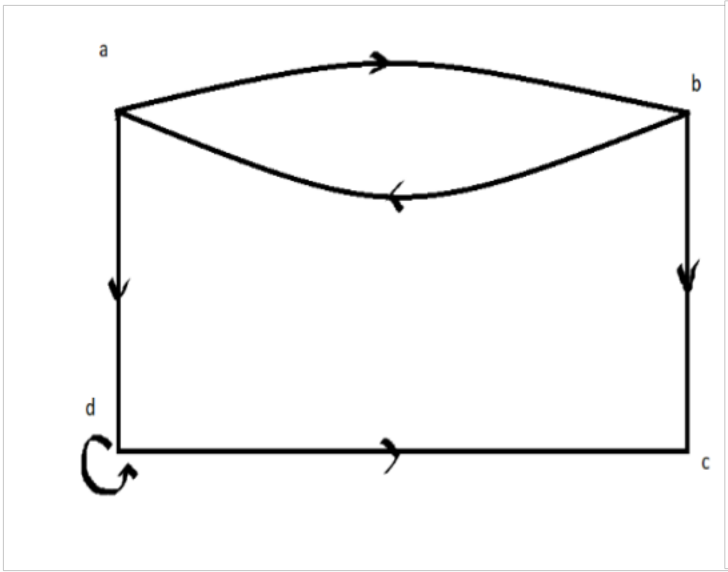
V	a	b	c	d	e	f	g	h
a	a	b	c	d	e	f	g	h
b	b	b	f	d	f	f	g	h
c	c	f	c	f	e	f	g	h
d	d	d	f	d	f	f	h	h
e	e	f	e	f	e	f	g	h
f	f	f	f	f	f	f	h	h
g	g	g	g	h	g	h	g	h
h	h	h	h	h	h	h	h	h

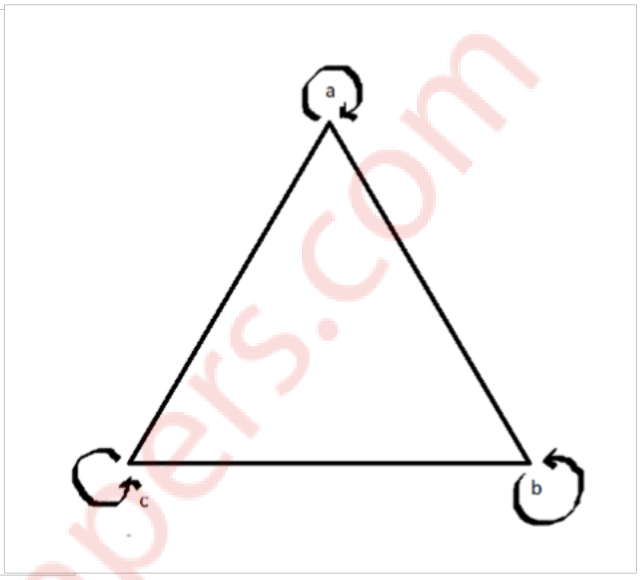
GLB:-

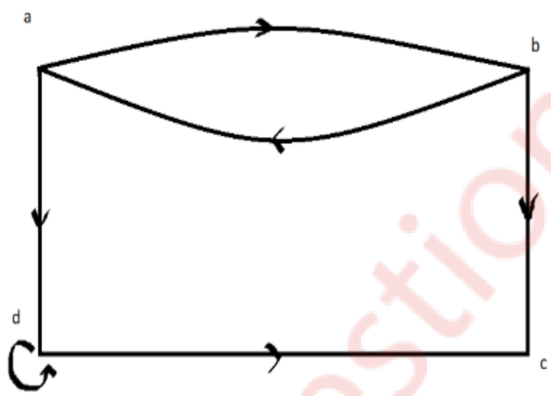
^	a	b	c	d	e	f	g	h
a	a	a	a	a	a	a	a	a
b	a	b	a	b	a	b	b	b
c	a	a	c	a	c	a	c	a
d	a	b	a	d	a	d	b	d
e	a	a	c	a	e	e	e	e
f	a	b	a	d	e	f	-	f
g	a	b	c	b	e	-	g	g
h	a	b	a	d	e	f	g	h



Q5.c) From the following digraphs, write the relation a set of ordered pairs. Are the relations equivalence relations? [4]





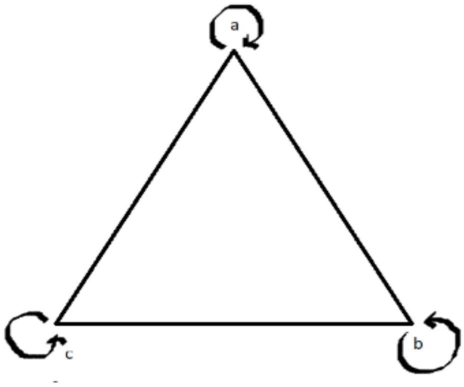


An equivalence relation is reflexive, symmetric and transitive.

R_1 = {(a,b), (a,c), (b,a), (b,d),(c,c),(c,d)}

R_1 is not reflexive because (a,a) ,(b,b), and (d,d) does not exist

R_1 is not equivalence.



R_2 = {(a,a), (b,b), (c,c), (a,b), (b,c), (c,a)}

Relation R_2 is reflexive and transitive but not symmetric because (a,b) exists but (b,a), (c,b) (a,c) do not belong to R_2

Hence R_2 is not equivalence relation.

Q5.d) For the set X = {2, 3, 6, 12, 24, 36}, a relation ≤ is defined as x≤y if x divides y. Draw the Hasse diagram for (X, ≤). Answer the following:

i)What are the maximal and the minimal elements?

ii)Give one example of chain and antichain

iii)Is the poset a lattice. [4]

Solution:-

$$R = \begin{Bmatrix} (2,2), (2,6), (2,12), (2,24),(2,36), (3,3), (3,6), (3,12), (3,24), (3,36), (6,6), (6,12) \\ (6,24), (6,36), (12,12), (12,24), (12,36), (24,24), (36,36) \end{Bmatrix}$$

Hasse diagram:-

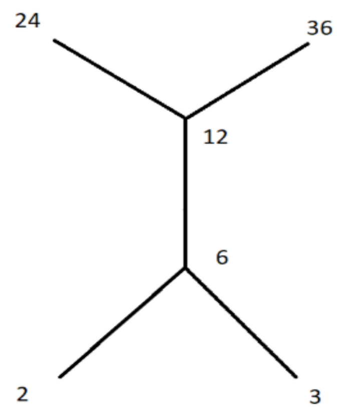
1. Maximal : 24,36

   Minimal : 2,3

2. Chain = {2,6,12,24}

   Antichain ={2,3}

   This poset is not a lattice



Q6.a) Prove that the set (1, 2, 3, 4, 5, 6} is a group under multiplication modulo 7. **[6]**

Solution:-

Multiplication module 7 table for set A is

X_7	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

1 is identity element of algebraic system

a X_7 1 = a = 1 X_7 a

eg:- 1 X_7 1 = 1; 2 X_7 1 = 2 ; 3 X_7 1 = 3 ………………. 6 X_7 1 = 6

recall that a^{-1} is that element of G such that a * a^{-1}

2 X_7 4 = 1 inverse of 2 = 4

3 X_7 5 = 1 inverse of 3 = 5

6 X_7 6 = 1 inverse of 6 = 6

We have $2'$ = 2

$2^2 = 2 \ X_7 \ 2 = 4$

$2^3 = 2 \ X_7 \ 2 = 4 \ X_7 \ 2 = 1$

$2^4 = 8 \ X_7 \ 2 = 1 \ X_7 \ 2 = 2$

Hence |2| = 3

2 is not generator

We have $3^{'} = 3$

$3^2 = 3 \ X_7 \ 3 = 2$

$3^3 = 9 \ X_7 \ 3 = 2 \ X_7 \ 3 = 6$

$3^4 = 27 \ X_7 \ 3 = 6 \ X_7 \ 3 = 4$

$3^5 = 3^4 \ X_7 \ 3 = 4 \ X_7 \ 3 = 5$

$3^6 = 3^5 \ X_7 \ 3 = 5 \ X_7 \ 3 = 1$

3 is generator of this group and is cyclic

Subgroup generated by {3,4} is denoted by <{3,4}> since 3,4 are element of this set they have to be there in <3,4>

Inverse of 3 is 5 and 4 is 2

3,4,5,2 ∈ <{3,4}>

3 $X_7$4 = 5 X_74 =5 — 5 X_7 4 = 6

3 X_7 3 = 2 X_7 3 = 2 — 6 X_7 6 = 1

3 X_7 5 = 1 X_7 5 =1 — 5 X_7 1 = 5

4 X_7 4 = 2 X_7 4 = 2 — 1 X_7 1 = 1

3 X_7 2 = 6 X_7 2 =6 — 5 X_7 2 = 3

5 X_7 5 = 4 — 3 X_7 6 = 4

5 X_7 6 = 2 — 2 X_7 2 = 4

<3,4> = <1,2,3,4,5,6>

Subgroup generated by <{3,4}> is the set A itself.

Hence the set (1, 2, 3, 4, 5, 6} is a group under multiplication modulo 7.

Q6.b) Given a generating function, find out corresponding sequence **[6]**

i) $\frac{1}{3-6x}$ ii) $\frac{x}{1-5x+6x^2}$

Solution:-

1. $\frac{1}{3-6x} = \frac{1}{3(1-2x)}$

The simple geometric function that gives the sum of geometric series

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

Replace x by 2x

$\frac{1}{1-2x} = \sum_{n=0}^{\infty} 2^n . x^n$

Multiply this by 1/3

$\frac{1}{3(1-2x)} = \frac{1}{3} \sum_{n=0}^{\infty} 2^n . x^n$

The associated sequence is $\left(0, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \ldots\ldots\right)$

2. $\frac{x}{1-5x+6x^2}$

We know : $1 - 5x + 6x^2 = (1 - 2x)(1 - 3x)$

Therefore $\frac{x}{1-5x+6x^2} = \frac{x}{(1-2x)(1-3x)}$

$x = A(1-3x) + B(1 - 2x)$

Put x = ½ and 1/3

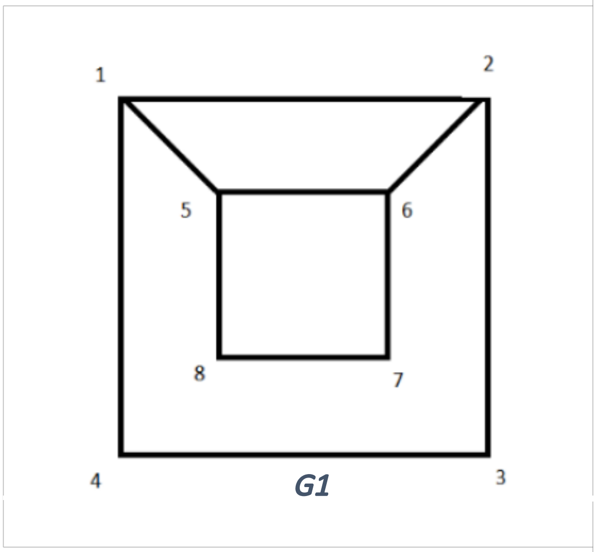
We get A = -1 and B = 1

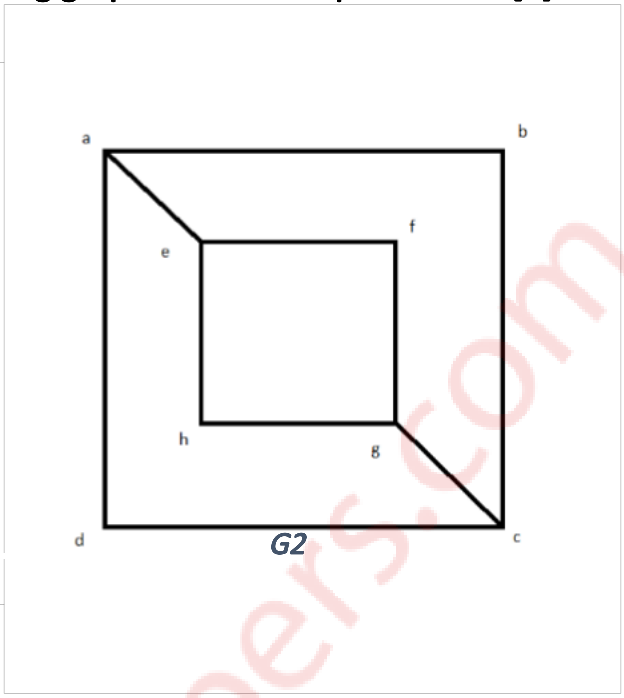
$f(x) = \frac{1}{(1-3x)} + \frac{1}{(1-2x)} = \sum_{n=0}^{\infty} (3^n x^n - 2^n x^n)$

$a_n = 3^n - 2^n$ for $n \geq 0$

Sequence is (0, 1, 5, 19, 65 ,.....)

Q6.c) Determine whether the following graphs are isomorphic or not.[4]





Solution:-

Hence both graph G_1 and G_2 contain 8 vertices and 10 edges no of vertices of degree 2 in both graphs are 4. The number of vertices of degree 3 in both graphs are 4.

For adjacency , consider the vertex 1 of degree 3. In G_1 it is adjacent to two vertices of degree 3 and 1 vertex of degree 2. But in G_2 there does not exist any vertex of degree 3 which is adjacent to degree 3 and 1 vertex of degree 2. Hence adjacency is not presented. Hence given graphs are not isomorphic.

Q6.d) Prove the following (use laws of set theory) [4]

$$A \times (X \cap Y) = (A \times X) \cap (A \times Y)$$

Solution:-

$A \times (X \cap Y) = (A \times X) \cap (A \times Y)$

Let (a,x) $\in A \times (X \cap Y)$ .... (1)

By definition of the Cartesian product

a $\in A$ and x $\in X \cap YY$

ince, x $\in X \cap Y$

x $\in X$ and x $\in Y$

(a,x) $\in A \times X$ and (a,x) $\in A \times Y$

$(a,x) \in (A \times X) \cap (A \times Y)$

$A \times (X \cap Y) \subseteq (A \times X) \cap (A \times Y)$ .... (2)

Again let; $(a,x) \in (A \times X)$ & $(a,x) \in (A \times Y)$

$a \in A$, $x \in X$ & $x \in Y$ Y

$\in$ AA & $x \in X \cap Y$

$(a,x) \in A \times (X \cap Y)$

$(A \times X) \cap (A \times Y) \subseteq A \times (X \cap Y)$ .... (3)

From (1) and (2) we get,

$$\mathbf{A \times (X \cap Y) = (A \times X) \cap (A \times Y)}$$

DISCRETE STRUCTURES

DEC 19 (CBCS)

Q1 a) Prove using Mathematical Induction

$1^2+2^2+3^2+\ldots\ldots n^2=n(n+1)(2n+1)/6$ **(5)**

Solution:

Let P(n)= $1^2+2^2+3^2+\ldots\ldots n^2=n(n+1)(2n+1)/6$

Step1: n=1

LHS=1^2

=1

RHS=1(1+1)(2x1+1)/6

=1(2)(3)/6

= 1

LHS=RHS

P(n) is true for n=1.

Step2:

Let P(n) be true for n=k

$1^2+2^2+3^2+\ldots\ldots k^2=k(k+1)(2k+1)/6$ ……………………………(1)

Now we have to prove that P(n) is true for n=k+1

$1^2+2^2+3^2+\ldots\ldots(k+1)^2=(k+1)(k+1+1)[2(k+1)+1]/6$

LHS= $1^2+2^2+3^2+\ldots\ldots k^2+(k+1)^2$

= $k(k+1)(2k+1)/6 +(k+1)^2$

=$(k+1)[k(2k+1)/6 + (k+1)]$

=$1/6\ (k+1)(2k^2+k+6k+6)$

=$1/6\ (k+1)(2k^2+7k+6)$

=$1/6\ (k+1)\ (k+2)\ (2k+3)$

RHS=$(k+1)(k+1+1)[2(k+1)+1]/6$

=$1/6\ (k+1)(k+2)(2k+3)$

LHS=RHS

P(n) is true for n=k+1

Hence from step1 and step2

By the principal of mathematical induction

$1^2+2^2+3^2+\ldots\ldots n^2=n(n+1)(2n+1)/6$

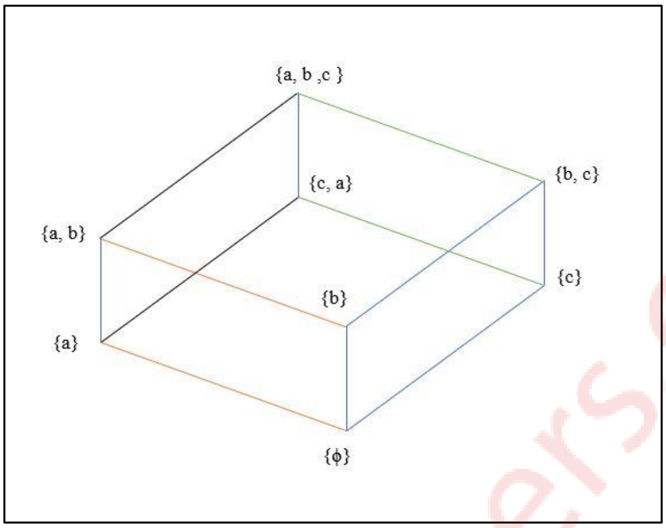
Q1 b) Let A={a, b, c } . Draw Hasse Diagram for {p(A), ⊆} **(5)**

Solution:

A= {a, b, c}

P(A)= {ϕ, {a}, {b}, {c}, {a,b},{b,c},{c,a},{a,b,c} }

Hasse Diagram is as follows:

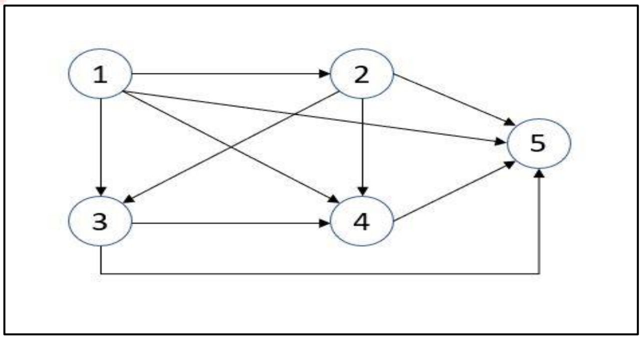


Q1 c)Let A={1,2,3,4,5}. A relation R is defined on A as aRb iff a<b. Compute R^2 and R ∞ **(5)**

Solution:

R={(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5)}

Diagraph of R is as shown below:



R^2:

$1R^23$ since 1R2 and 2R3

$1R^24$ since 1R2 and 2 R 4

$1R^25$ since 1R2 and 2R5

$2R^24$ since 2R3 and 3R4

$2R^25$ since 2R4 and 4R5

$3R^25$ since 3R4 and 4R5

R^2={(1,3),(1,4),(1,5),(2,4),(2,5),(3,5)}

R∞={(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5)}

Q1 d) Let f: R O R, where f(x)=2x-1 and $f^{-1}(x)=(x+1)/2$

Find $(f \text{ O } f^{-1})(x)$

Solution: **(5)**

f(x)=2x-1

$f^{-1}(x)=(x+1)/2$

$(f \text{ O } f^{-1})(x)= f(f^{-1}(x))$

$=f(\frac{x+1}{2}$

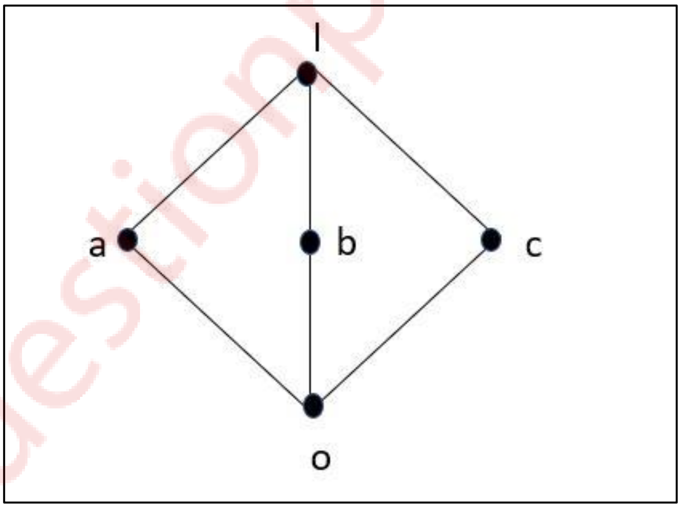
$= 2(\frac{x+1}{2})-1$

$=x+1-1$

$\mathbf{=x}$

$(f \circ f^{-1})(x)=x$

Q2 a) **Define Distributive lattice. Check if the following diagram is a distributive lattice or Not. (4)**



Solution:

A lattice L is called distributive if for any elements a, b and c in L we have the following distributive properties

1. a ∧ (b ∨ c) =(a ∧ b) ∨ (a ∧ c)

2. a ∨ (b ∧ c)=(a ∨ b) ∧ (a ∨ c)

From given figure :

a ∧ (b ∨ c) =a ∧ 1 = a

While,

(a ∧ b) ∨ (a ∧ c)= o ∨ o = o

Therefore given figure is **non-distributive.**

Q2 b) Prove that set G={1,2,3,4,5,6} is a finite abelian group of order 6 w.r.t multiplication module 7.

Solution: **(8)**

X_7	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

G1:

Consider any three numbers from table

5 x (6 x 3)=5 x 4=6

(5 x 6) x 3=2 x 3=6

As 5 x (6 x 3)=(5 x 6) x 3

Hence x is associative.

G2:

From table we observe first row is same as header.

I €G

Hence Identity of x exists.

G3:

Consider any two number from table

4 x 2 =1 and 2 x 4=1

Hence x is commutative.

G4:

Inverse of x exists.

Q2 c) Find the number of positive integers not exceeding 100 that are not divisible by 5 or 7. Also draw corresponding venn diagram. (8)

Solution:

Let

A: All positive integers not exceeding 100

A1: Divisible by 5

A2:Divisible by 7

There are 100 integers not exceeding 100

|A|=100

There are 100 integers not exceeding 100, while a number divisible by5 is every 5^{th} element in the lost of positive integers. Use the division rule:

|A1|= $\frac{|A|}{d}$ =100/5=20

Similarly we obtain for numbers divisible by 7 (round down)

|A2|= $\frac{|A|}{d}$ =100/7=14

Numbers divisible by 5 and 7 are divisible by 35(round down)

|A1 U A2|= $\frac{|A|}{d}$ =100/35=2

By principal of inclusion-exclusion

|A1 U A2|= |A1|+ |A2|- |A1 ∩ A2|

= 20+14-2

= 32

Thus 32 of the 100 integers are divisible by 5 or 7 , then the number of integers not divisible by 5 or 7 are

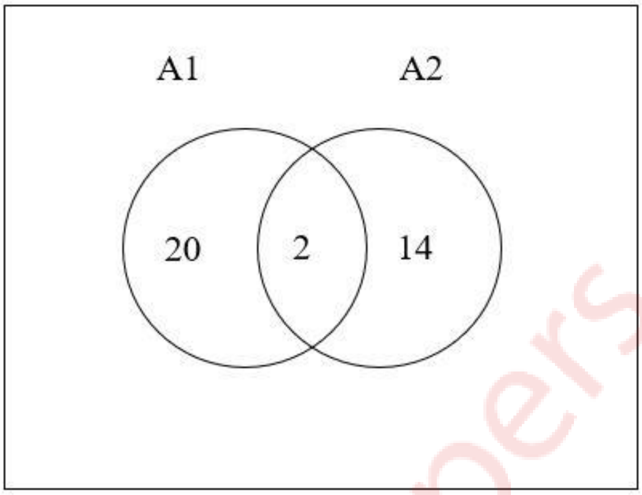
|(A1 U A2)^{C}|=|A|-|A1 U A2|

= 100 - 32

= 68

Thus, there are 68 integers not divisible by 5 or 7.

Venn Diagram:



Q3 a) Construct Truth Table and check if the following statement is tautology.

$(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$

Solution: **(4)**

P	**Q**	**$((P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P))$**
F	F	**T**
F	T	**T**
T	F	**T**
T	T	**T**

Hence the given statement is tautology.

Q3 b) Consider the (2,5) group encoding function defined by **(8)**

e(00)=00000

e(01)=01110

e(10)=10101

e(11)=11011

Decode the following words relative to maximum likelihood decoding functions.

i)11110 ii)10011 iii)10100

Solution:

The decoding table is as follows:

00000	01110	10101	11011
00001	01111	10100	11010
00010	01100	10111	11001
00100	01010	10001	11111
01000	00110	11101	10011
10000	11110	00101	01011

1) Encoded word= 11110

Corresponding encode word belongs to the column 01110

Therefore d(11110)=01

2) Encoded word= 10011

Corresponding encode word belongs to the column 11011

Therefore d(01010)=11

3) Encoded word= 10100

Corresponding encode word belongs to the column 10101

Therefore d(00110)=10

Q3c)How many four digits can be formed out of digits 1,2,3,5,7,8,9 if no digits repeated twice? How many of these will be greater than 3000?

Solution: **(8)**

We have to make 4 digit number without repetition using 1,2,3,5,7,8,9

For this we have to fill 4 spaces (_ _ _ _) with required numbers.

1st space can be filled in 7 ways. (7 _ _ _)

2nd space can be filled in 6 ways because we already used one digit in previous space so only 6 digits are remaining now. (7 6 _ _)

Similarly 3rd and 4th space can be filled in 5 and 4 respectively. (7 6 5 4)

So the no of four digits can be formed out of 1,2,3,5,7,8,9 =7*6*5*4

=840 digits

The four digit number greater than 3000 are:

The first place can have number 3,5,7,8,9 i.e 5 digits. 1st space can be filled in 5 ways.

(5 _ _ _)

2nd space can be filled in 6 ways because we already used one digit in previous space so only 6 digits are remaining now. (5 6 _ _)

Similarly 3rd and 4th space can be filled in 5 and 4 respectively. (5 6 5 4)

So the no of four digits can be formed out of 1,2,3,5,7,8,9 which are greater than 3000 are =5*6*5*4

=600 digits

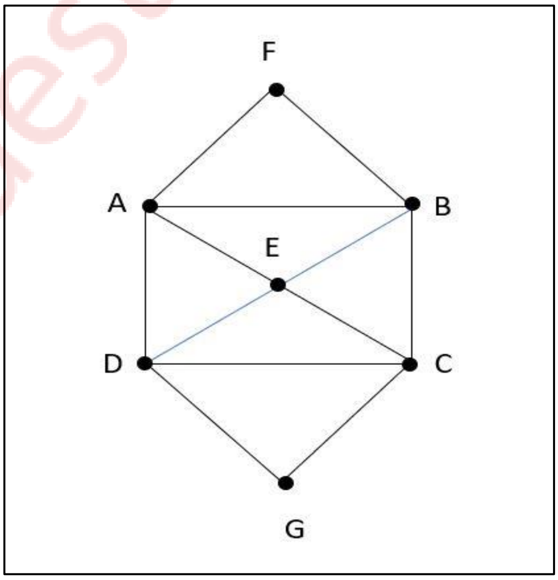
Q4a) A bag contains 10 red marbles, 10 white marbles and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same color?

Use pigeonhole Principle.

Solution: **(4)**

Apply pigeonhole principle.
No. of colors (pigeonholes) n = 3
No. of marbles (pigeons) K+1 = 4
Therefore the minimum no. of marbles required = Kn+1
By simplifying we get Kn+1 = 10.
Verification: ceil[Average] is [Kn+1/n] = 4
[Kn+1/3] = 4
Kn+1 = 10
i.e., 3 red + 3 white + 3 blue + 1(red or white or blue) = 10

Q4b) Define Euler path, Euler circuit, Hamiltonian path and Hamiltonian circuit. Determine if following diagram has Euler path, Euler circuit, Hamiltonian path and Hamiltonian circuit and state the path/circuit. **(8)**



Solution:

Euler path: An Euler path is a path that uses every edge of a graph exactly once.

Euler circuit: An Euler circuit is a circuit that uses edges of a graph exactly once and which starts and ends with same vertex.

Criteria for Euler cycle:

If a connected graph G has a Euler circuit, then all vertices Of G must have a even degree

Criteria for Euler path:

If a connected graph G has a Euler path then it must have exactly two vertices with odd degree. The two endpoints of Euler path must be the vertices with odd degree.

In above graph all the vertex has even degree.

Hence there is no Euler path but it has Euler circuit.

Euler circuit: **BCEADGCDEBAFB**

Hamilton path: Hamiltonian path is a graph that visits each vertex exactly once.

Hamiltonian path: **FABEDCG**

Hamiltonian circuit: Hamiltonian circuit is a path that visits every vertex exactly once and which starts and ends on the same vertex.

Criteria for Hamiltonian circuit:

The given condition are necessary but not sufficient

A) A simple graph with n vertices (n>=3) is Hamiltonian if every vertex has degree n/2 or greater.

B) A graph with n vertices (n>=3) is Hamiltonian if for every pair of non-adjacent , the sum of their degrees is n or greater.

Hamiltonian circuit: **FADGCEBF**

Q4c) In how many ways a committee of three faculty members and 2 students can be formed from 7 faculty members and 8 students. (8)

Solution:

A committee of 3 faculty and 2 students need to be formed.

Available faculty and students are 7 and 8 respectively.

Out of 7 faculty members 3 faculty members can be chosen in 7C3 ways.

Out of 8 students 2 students can be chosen in 8C2 ways.

Total number of ways of forming a committee =(7C3) * (8C2)

=(7 * 6 * 5 / 1 * 2 * 3) x (8 * 7 / 1 * 2)

=980 ways.

We can form a committee of three faculty members and 2 students from 7 faculty and 8 students in 980 ways.

Q5a)Let Z_n denote the set of integers {0,1,2,…….n-1}. Let Θ be a binary operation on Z_n such that a Θ b=reminder of ab divided by n

I) Construct table for the operation Θ for n=4

II) Show that (Z_n,Θ) is a semi group for any n

Solution: (4)

i)The table for the operation Θ for n=4

Θ	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

ii) The set Z_n is closed under the operation ʘ because for any

a,b Є Z_n , a ʘ b Є Z_n ……………………..1)

Now check for associativity for any a,b,cЄ Z_n

(a $ʘ_4$b) $ʘ_4$c = a $ʘ_4$(b $ʘ_4$c)

Let a=1,b=2,c=3

(1 $ʘ_4$2) $ʘ_4$3 = 1 $ʘ_4$(2 $ʘ_4$3)

2 $ʘ_4$3=1 $ʘ_4$2

2=2

∴ ʘ is an associative operation. ……………………….2)

From 1) and 2) we conclude that (Z_n,ʘ) is a semigroup for any 'n'.

Q5b)Find Transitive Closure of R represented by M_R as follows Using Warshall's algorithm set {a,b,c,d}

$$M_R=\begin{vmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

Solution: **(8)**

$$W= \begin{matrix} & a & b & c & d \\ a & 0 & 1 & 0 & 1 \\ b & 1 & 0 & 1 & 0 \\ c & 1 & 0 & 0 & 0 \\ d & 0 & 0 & 0 & 0 \end{matrix}$$

Step1: First we copy all 1's from W to matrix W1.

We observe W has 1's at b and c position in first column and has 1's at b and d in first row.

So add 1's at (b,b),(b,d),(c,b),(c,d)

$\therefore$W1 is same as W

$$W1= \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{matrix} a & b & c & d \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Step 2: We copy all 1's from W1 to W2.

We observe that W1 has 1's at (a,b,c) position in second column and at(a,b,c,d) position in second row. So we add 1's at (a,a),(a,b),(a,c),(a,d),(b,a),(b,b),(b,c),(b,d),(c,a),(c,b),(c,c),(c,d) position in W2.

$$W2= \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{matrix} a & b & c & d \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Step 3:we copy all 1's from W2 to W3.

We observe that W2 has 1's at (a,b,c) position in third column and (a,b,c,d) position in third row

So W2 and W3 are same.

$$W3 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

<u>Step4:</u> First we copy all 1's from W3 to matrix W4.

We observe W3 has 1's at (a,b,c) position in fourth column and has no 1's at fouth row.

$\therefore$W3 is same as W4

$$W4 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Hence by Warshall's Algorithm

Transitive closure={(a,a),(a,b),(a,c),(a,d),(b,a),(b,b),(b,c),(b,d),(c,a),(c,b),(c,c),(c,d)}

Q5c) Let A={1,2,3,4,5} and let

R={(1,1),(1,3),(1,4),(2,2),(2,5),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4),(5,2),(5,5)}. Check if R is a equivalence relation. Justify your answer . Find equivalence classes of A.

Solution: **(8)**

Equivalence relation: A relation R on set A is called equivalence relation if it is reflexive, symmetric and transitive.

A={1,2,3,4,5}

R={(1,1),(1,3),(1,4),(2,2),(2,5),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4),(5,2),(5,5)}

R is reflexive since (1,1),(2,2),(3,3),(4,4),(5,5) € R.

R is symmetric.

R is transitive.

Hence given relation is equivalence relation.

The equivalence classes of elements A are:

[1]={1,3,4}

[2]={2,5}

[3]={1,3,4}

[4]={1,3,4}

[5]={2,5}

Q6a)How many vertices are necessary to construct a graph with exactly 6 edges in which each vertex is of degree 2.

Solution: **(4)**

Handshaking Lemma: The sum of the degrees of all vertices of any graph is equal to twice number of edges.

Here, number of edges (e) =6

Degree of each vertex= d(v) =2

As per the lemma $\sum_{i=1}^{n} d(V\text{i}) = 2e$

∴ n x 2= 2x6

∴ n=6

Hence, there are 6 vertices in the graph.

Q6b)What is the solution of the recurrence relation $a_n = -a_{n-1} + 4a_{n-2} + 4a_{n-3}$ with $a_0=8, a_1=6$ and $a_2=26$?

Solution: **(8)**

Since it is a linear homogeneous recurrence, we find the characteristic equation of:

$r^3 + r^2 - 4r - 4 = 0$

which we can rewrite as:

$r^3 + -4r + r^2 - 4 = r(r^2 - 4) + (r^2 - 4) = 0$

which factors as (r + 1)(r − 2)(r + 2).

From our theorem, we know that

$an = \alpha 1(-1)^n + \alpha 2(-2)^n + \alpha 3\,(2^n)$ is a solution.

We need to use the initial conditions to solve this.

$a0 = \alpha 1 + \alpha 2 + \alpha 3 = 8$ …………………..(i)

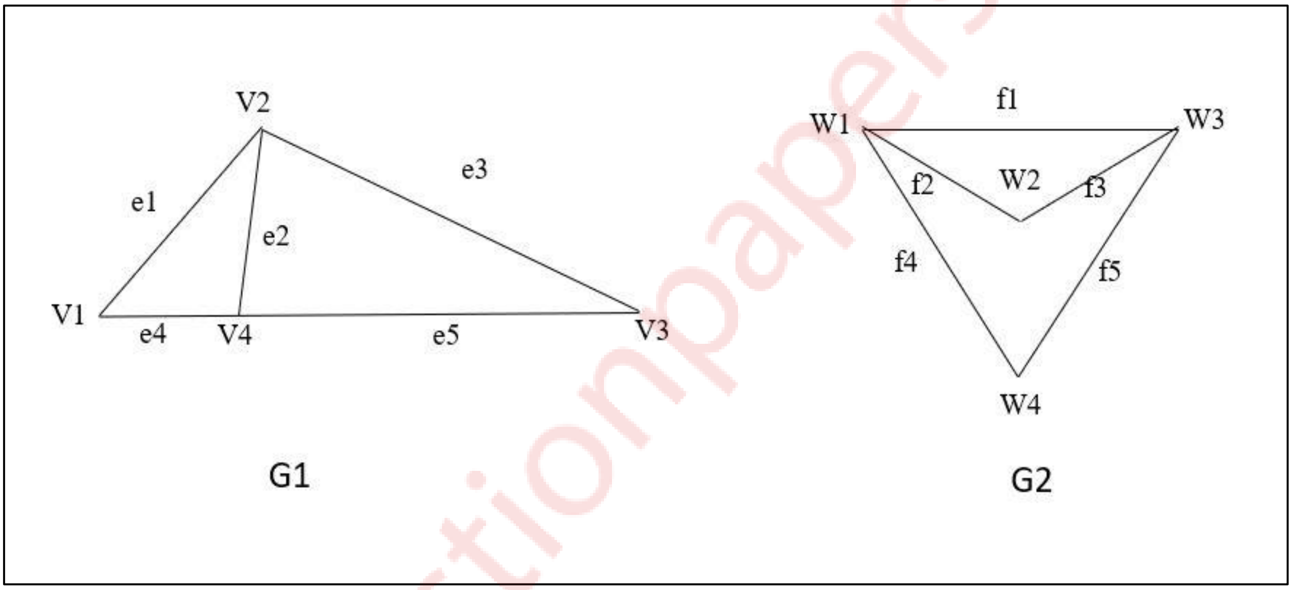
$a1 = -\alpha 1 - 2\alpha 2 + 2\alpha 3 = 6$ …………………..(ii)

$a2 = \alpha 1 + 4\alpha 2 + 4\alpha 3 = 26$ ………………….(iii)

After multiplying the first equation by 4 and then subtracting the third equation, we get $\alpha 1 = 2$. Substituting and combine the second and third equations, we get $\alpha 2 = 1$, and then $\alpha 3 = 5$.

Thus the closed form of the recurrence is: an = $2 \quad (-1)^n + (-2)^n + 5 \text{ x } (2^n)$.

Q6c) Determine if following graphs G1 and G2 are isomorphic or not. (8)



Solution:

Two graphs G(V,E) and H(V',E') isomorphic if

1) There is one to one correspondence f from V to V' such that f(V1)=V1' and f(V2)=V2' for every (v1, v2)€V and (v1', v2')€ V'
2) G and H should have equal no. Of edges.
3) G and H should have equal no. Of vertices.
4) G and H should have same degree of vertices
5) Adjacency property is observed in each vertex.

From the above two graphs:

Graph G1		
Number of vertices	4	
Number of Edges	5	
Vertex	**Degree of vertex**	**Adjacent vertices**
V1	2	V2(3),V4(3)
V2	3	V1(2),V4(3),V3(2)
V3	2	V2(3),V4(3)
V4	3	V1(2),V2(3),V3(2)

Graph G2		
Number of vertices	4	
Number of Edges	5	
Vertex	**Degree of vertex**	**Adjacent vertices**
W1	3	W2(2),W3(3),W4(2)
W2	2	W1(3),W3(3)
W3	3	W1(3),W2(2),W4(2)
W4	2	W1(3),W3(3)

We observe that,

There are equal no of edges and vertices for both graph.

Graph G1 has 2 vertices with degree 2, two vertices with degree 3.

Graph G2 has 2 vertices with degree 2, two vertices with degree 3.

Adjacency property is observed in each vertex.

Hence the two graphs are isomorphic.

DISCRETE STRUCTURES
MAY 19 (CBCS)

Q1 a) Prove using Mathematical Induction
2+5+8+….+(3n-1)=n(3n+1)/2 **(5)**

Solution:
Let P(n)= 2+5+8+….+(3n-1)=n(3n+1)/2

Step1: n=1
LHS=3x1-1
=2

RHS=1(3x1+1)/2
=2
LHS=RHS
P(n) is true for n=1.

Step2:
Let P(n) be true for n=k
2+5+8+….+(3k-1)=k(3k+1)/2 ……………………………….(1)

Now we have to prove that P(n) is true for n=k+1

2+5+8+….(3k-1)+[3(k+1)-1]=(k+1)[3(k+1)+1]/2
LHS= 2+5+8+….(3k-1)+[3(k+1)-1]
=k(3k+1)/2+[3(k+1)-1]
=$(3k^2+k)/2+[3k+2]$
=$3k^2+7k+4/2$
=(k+1)(3k+4)/2

RHS=(k+1)[3(k+1)+1]/2
=(k+1)(3k+4)/2

LHS=RHS
P(n) is true for n=k+1

Hence from step1 and step2
By the principal of mathematical induction
2+5+8+….+(3n-1)=n(3n+1)/2

Q1 b) Find the generating function for the following finite sequences

I) 1,2,3,4,……….

II) 2,2,2,2,2 **(5)**

Solution:

I) If $\{a_m\}=\{a_0,a_1,a_2,a_3….\}$ is a sequence of real numbers and x is a real variable then Ordinary generating function of the sequence is infinite sum

$$g(x)= \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + ……..$$

For sequence $\{a_n\}=\{1,2,3,4,…….\}$

$g(x)= 1+2x + 3x^2 + 4x^3 +……..$

$=(1-x)^{-2}$

$=1/(1-x)^2$

The generating function $g(x) = 1/(1-x)^2$

II) If $\{a_m\}=\{a_0,a_1,a_2,a_3….\}$ is a sequence of real numbers and x is a real variable then Ordinary generating function of the sequence is infinite sum

$$g(x)= \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + ……..$$

For sequence $\{a_n\}=\{2,2,2,2,2\}$

$g(x)= 2+2x + 2x^2 + 2x^3 +2x^4$ which id GP with first term a=2 number of terms n=5 and common ration r=x

In GP sum of series

$$S_n = \frac{a}{r-1}(r^n - 1)$$

The generating function $g(x) = \frac{2}{x-1}(x^5 - 1)$

Q1 c) Let A={1,4,7,13} and R={(1,4), (4,7), (7,4),(1,13)}

Find Transitive closure using Warshall's Algorithm. **(5)**

Solution:

Relation matrix W= M_R=

	1	4	7	13
1	0	1	0	1
4	0	0	1	0
7	0	1	0	0
13	0	0	0	0

Step1: First we copy all 1's from W to matrix W1.

We observe W has no 1's at first column and has 1's at 4 and 13 in first row.

$\therefore$ W1 is same as W

$$W1 = \begin{array}{c} \\ 1 \\ 4 \\ 7 \\ 13 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 4 & 7 & 13 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Step 2: We copy all 1's from W1 to W2.

We observe that W1 has 1's at (1,7) position in second column and at(7) position in second row. So we add 1's at (1,7) and (7,7) position in W2.

$$W2 = \begin{array}{c} \\ 1 \\ 4 \\ 7 \\ 13 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 4 & 7 & 13 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Step 3:we copy all 1's from W2 to W3.

We observe that W2 has 1's at (1,4,7) position in third column and (4,7) position in third row

So we add 1's at (1,4), (1,7),(4,4),(4,7),(7,4)(7,7) in W3.

$$W3 = \begin{array}{c} \\ 1 \\ 4 \\ 7 \\ 13 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 4 & 7 & 13 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Step4: First we copy all 1's from W3 to matrix W4.

We observe W3 has 1's at (13) position in fourth column and has no 1's at fouth row.

$\therefore$ W3 is same as W4

$$W4 = \begin{array}{c|cccc} & 1 & 4 & 7 & 13 \\ \hline 1 & 0 & 1 & 1 & 1 \\ 4 & 0 & 1 & 1 & 0 \\ 7 & 0 & 1 & 1 & 0 \\ 13 & 0 & 0 & 0 & 0 \end{array}$$

Hence by Warshall's Algorithm

Transitive closure={(1,4)(1,7)(1,13)(4,4)(4,7)(7,4)(7,7)}

Q1 d) Let f: R O R, where f(x)=2x-1 and $f^{-1}(x)=(x+1)/2$

Find $(f \text{ O } f^{-1})(x)$

Solution: **(5)**

f(x)=2x-1

$f^{-1}(x)=(x+1)/2$

$(f \text{ O } f^{-1})(x)= f(f^{-1}(x))$

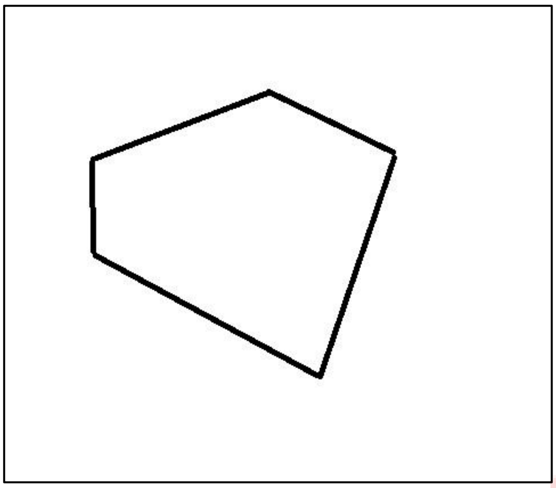
$=f(\frac{x+1}{2})$

$= 2(\frac{x+1}{2})-1$

=x+1-1

=x

Q2 a) Define lattice. Check if the following diagram is a lattice or not. (4)



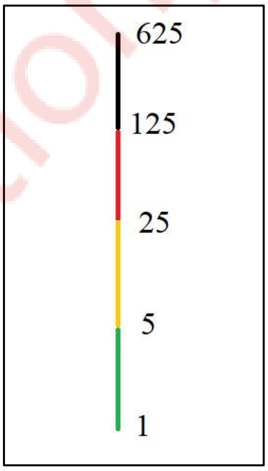
Solution:

A posset (L,≤) in which every pair (a,b) of L has a LUB(least Upper Bound) and GLB (Greatest Lower bound) is called a lattice.

For eg.

Let R be a relation of divisibility. Consider the set of divisors of 625

i. e. D_{625}={1,5,25,125,625}



The hasse diagram for D_{625} is

We observe that every pair of elements of D_{625} has a LUB and GLB.

Also each LUB and GLB ε D_{625}.

The given diagram is lattice because it has LUB and GLB.

Q2 b) Prove that set G={1,2,3,4,5,6} is a finite abelian group of order 6 w.r.t multiplication module 7.

Solution: **(8)**

X_7	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

G1:

Consider any three numbers from table

5 x (6 x 3)=5 x 4=6

(5 x 6) x 3=2 x 3=6

As 5 x (6 x 3)=(5 x 6) x 3

Hence x is associative.

G2:

From table we observe first row is same as header.

I $\in$ G

Hence Identity of x exists.

G3:

Consider any two number from table

4 x 2 =1 and 2 x 4=1

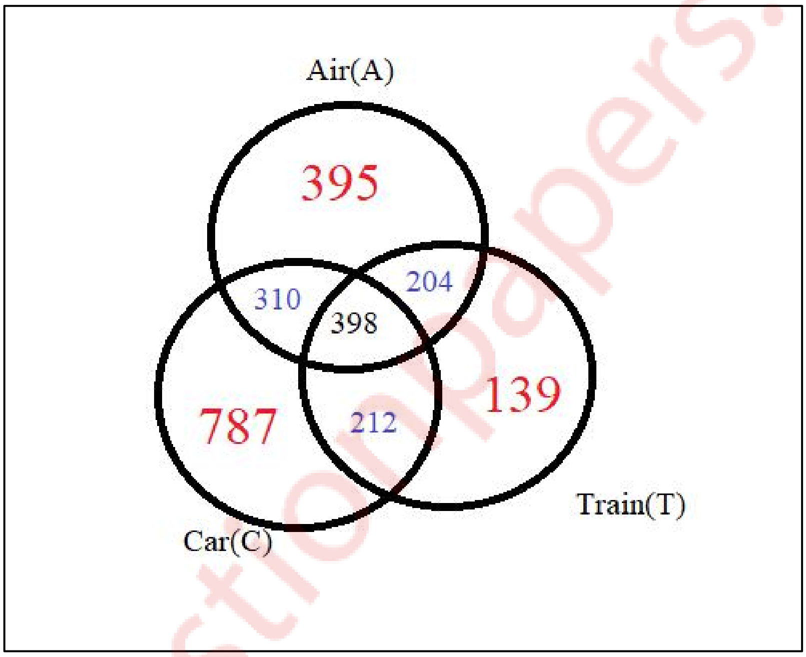
Hence x is commutative.

G4:

Inverse of x exists.

Q2 c) A travel company surveyed it's travelers, to learn how much of their travel is taken with an Airplane, a Train or a car. The following data is known; make a complete Venn Diagram with all the data. The number of people who flew was 1307. The number of people who both flew and used a train was 602. The people who used all three were 398 in number. Those who flew but didn't drive came to total 599. Those who drove but did not use train totaled 1097. There were 610 people who used both trains and cars. The number of people who used either a car or train or both was 2050. Lastly, 421 people used none of these .Find out how many people drove but used neither a train nor an airplane, and also, how many people were in the entire survey.

Solution: **(8)**



Let A be the set of people who flew
Let C be the set of people who travelled by car
Let T be the set of people who travelled by train
$N(A \cap T \cap C) = 398$

$N(A) = 1307$

$N(A \cap T) = 602$
The people who flewed and travelled by train but didn't travelled by car
= $N(A \cap T) - N(A \cap T \cap C)$
= 602-398

= 204

People who flew but didn't drive are 599.
Therefore people who only flew are 599- 204=395

N(C)= 1097
N(C∩T)=610
The people who travelled by car and train but didn't flew
= N(C∩T) - N(A∩T∩C)
=610-398
= 212

The people who flewed and travelled by car but didn't travelled by train are =310

Those who drove but did not use train totaled 1097
The people who travelled by only car = 1097-310=787
The people who travelled by only train= 139

The no of people who drove but used neither a train nor an airplane= **787**
The no of people in entire survey were =**2866**

Q3 a) Prove $\neg(p \vee (\neg p \wedge q))$ and $(\neg p \wedge \neg q)$ are logically equivalent by developing a series of logical equivalences.
Solution: **(4)**

$\neg(p \vee (\neg p \wedge q))$

$\neg p \wedge \neg(\neg p \wedge q)$	By Demorgan's law
$\neg p \wedge (p \vee \neg q)$	By double negation
$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$	By distributive law
$F \vee (\neg p \wedge \neg q)$	$(\neg p \wedge p)= F$
$(\neg p \wedge \neg q)$	By Identity law

Hence $\neg(p \vee (\neg p \wedge q))$ and $(\neg p \wedge \neg q)$ are logically equivalent.

Q3 b) Consider the (3,5) group encoding function defined by **(8)**
e(000)=00000
e(010)=01001
e(100)=10011
e(110)=11010
e(001)=00110
e(011)=01111

e(101)=10101
e(111)=11000
Decode the following words relative to maximum likelihood decoding functions.
i)11001 ii)01010 iii)00111

Solution:
The decoding table is as follows:

00000	00110	01001	01111	10011	10101	11010	11000
00001	00111	01000	01110	10010	10100	11011	11001
00010	00100	01011	01101	10001	10111	11000	11010
00100	00010	01101	01011	10111	10001	11110	11100
01000	01110	00001	00111	11011	11101	10010	10000
10000	10110	11001	11111	00011	00101	01010	01000
10001	10111	11000	11110	00010	00100	01011	01001
10010	10100	11011	11101	00001	00111	01000	01010

1) Encoded word= 11001
Corresponding encode word belongs to the column 01001
Therefore d(11001)=010

2) Encoded word= 01010
Corresponding encode word belongs to the column 11010
Therefore d(01010)=110

3) Encoded word= 00111
Corresponding encode word belongs to the column 00110
Therefore d(00110)=001

Q3c) Mention all the elements of set D_{36} also specify R on D_{36} as aRb if a|b. Mention Domain and range of R. Explain if the relation is equivalence relation or a Partially Ordered Relation. If it is A Partially Ordered Relation, draw its Hasse Diagram.
Solution: **(8)**

D_{36}={1,2,3,4,6,9,12,18,36}

R={(1,1)(1,2)(1,3)(1,4)(1,6)(1,9)(1,12)(1,18)(1,36)(2,2)(2,3)(2,4)(2,6)(2,9)(2,12)(2,18)(2,36)(3,3)(3,4)(3,6)(3,9)(3,12)(3,18)(3,36)(4,4)(4,6)(4,9)(4,12)(4,18)(4,36)(6,6)(6,9)(6,12)6,18)(6,36)(9,9)(9,12)(9,18)(9,36)(12,12)(12,18)(12,36)(18,18)(18,36)(36,36)}

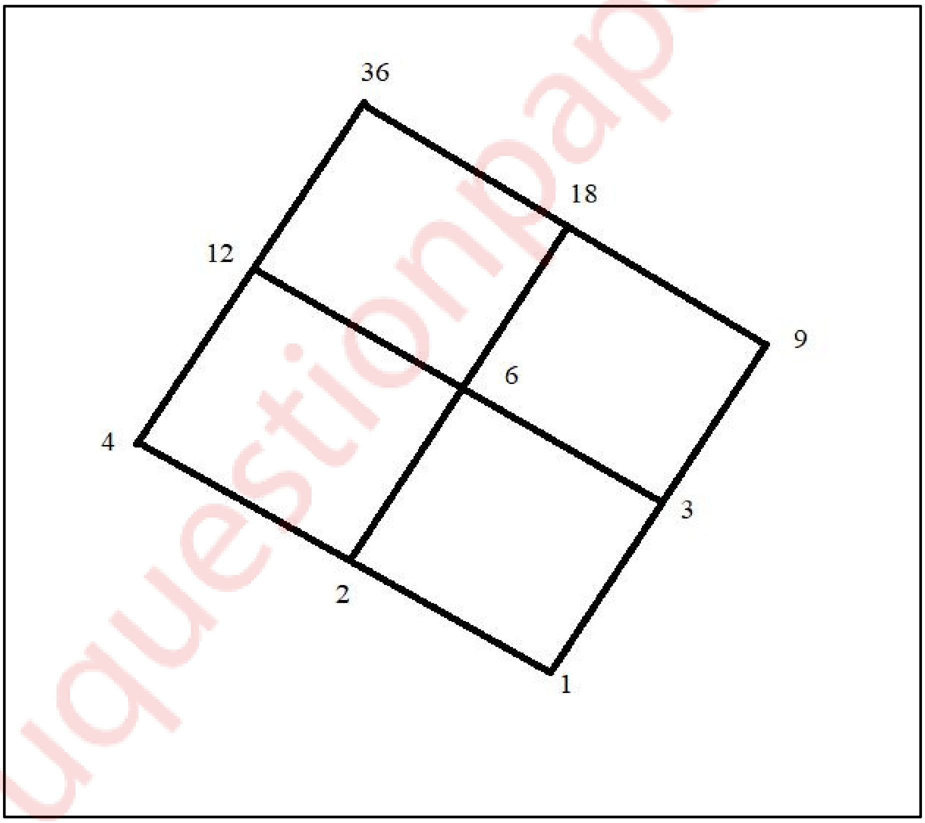
Domain of R ={1,2,3,4,6,9,12,18,36}
Range of R={1,2,3,4,6,9,12,18,36}

A relation R on a set A is called a partial order relation if it satisfies the following three properties:

- Relation R is Reflexive, i.e. aRa $\forall$ a$\in$A.
- Relation R is Antisymmetric, i.e., aRb and bRa $\Longrightarrow$a = b.
- Relation R is transitive, i.e., aRb and bRc $\Longrightarrow$aRc.

For D36 Relation R is reflexive, antisymmetric and transitive
Hence it is partial order relation.

Hasse diagram of D36 is



Q4a) Explain Extended pigeonhole principle. How many friends must you have to guarantee that at least five of them should will have birthdays in the same month.

Solution: **(4)**

Extended pigeonhole principal

It states that if n pigeons are assigned to m pigeonholes (The number of pigeons is very large than the number of pigeonholes), then one of the pigeonholes must contain at least [(n-1)/m]+1 pigeons.

Here Number of pigeons = n = ?

No. of pigeonholes = m = 12 (months)

∴ $[(n-1)/m]+1=5$

$[(n-1)/12]+1=5$

$n - 1 = 48$

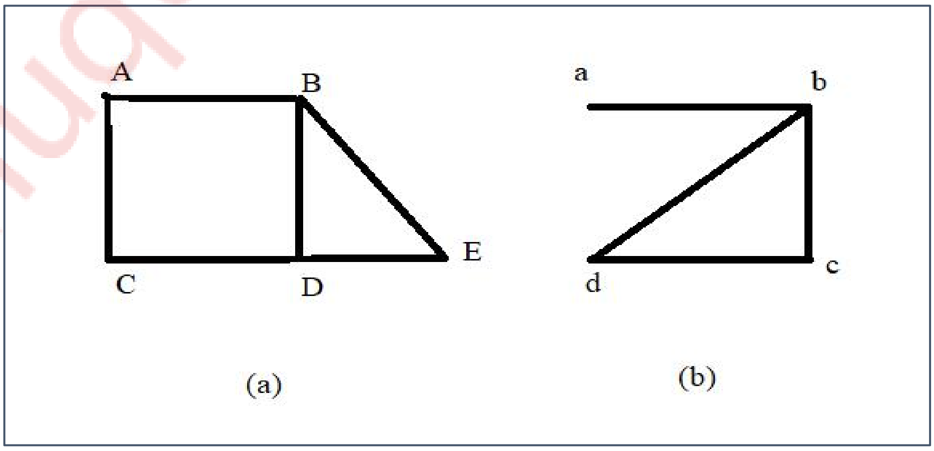
$n = 49$ [No. of pigeons]

∴ 49 friends should be their to guarantee that at-least five of them must have birthday in a same month of year.

Q4b) Define Euler path and Hamilton path.

I) Determine Euler cycle and path in graph shown in (a)

II) Determine Hamiltonian cycle and path in graph shown in (b) **(8)**



Solution:

Euler path: An Euler path is a path that uses every edge of a graph exactly once.

Euler circuit: An Euler circuit is a circuit that yses edges of a graph exactly once and which starts and ends with same vertex.

Criteria for Euler cycle:

If a connected graph G has a Euler circuit,then all vertices Of G must have a even degree

In fig(a)

Since vertices B and D have odd degree.

Therefore there is no Euler cycle for fig(a).

Criteria for Euler path:

If a connected graph G has a Euler path then it must have exactly two vertices with odd degree. The two endpoits of Euler path must be the vertices with odd degree.

The Euler path- BACDBED

Hamilton path: Hamiltonian path is a graph that visits each vertex exactly once.

Hamiltonian circuit: Hamiltonian circuit is a path that visitsevery vertex exactly once and which starts and ends on the same vertex.

Criteria for Hamiltonian circuit:

The given condition are necessary but not sufficient

A) A simple graph with n vertices (n>=3) is Hamiltonian if every vertex has degree n/2 or greater.

B) A graph with n vertices (n>=3) is hamiltonian if for every pair of non-adjacent , the sum of their degrees is n or greater.

For graph (b)

Hamiltonian path= abcdb

There is no hamiltonian cycle.

Q4c) In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

Solution: **(8)**

In a group of 6 boys and 4 girls, four children are to be selected such that
at least one boy should be there.
So we can have

(four boys) or (three boys and one girl) or (two boys and two girls) or (one boy and three girls)

This combination question can be solved as

$=({}^{6}C_{4})+({}^{6}C_{3}*\ {}^{4}C_{1})+({}^{6}C_{2}*\ {}^{4}C_{2})+({}^{6}C_{1}*\ {}^{4}C_{3})$

$=[6\times5/2\times1]+[(6\times5\times4/3\times2\times1)\times4]+[(6\times5/2\times1)(4\times3/2\times1)]+[6\times4]$

$=15+80+90+24$

=209

Q5a) Let G be a group. Prove that the identity element e is unique.

Solution: **(4)**

As the identity element $e\in G$ is defined such that $ae=a\ \forall a\in G$.

While the inverse does exist in the group and multiplication by the inverse element gives us the identity element, which assumes that the identity element is unique.

A more standard way to show this is suppose that e,f are both the identity elements of a group GG.

Then, e= e o f since f is the identity element.

=f since e is the identity element.

This shows that the identity element is indeed unique.

Q5b) A pack contains 4 blue, 2 red and 3 black pens. If 2 pens are drawn at random from the pack, Not replaced and then another pen is drawn. What is the probability of drawing 2 blue pens and 1 black pen?

(8)

Solution:

Total blue pens: 4

Total red pens: 2

Total black pens: 3

Total pens: 4+2+3= 9

Probability of drawing 2 blue pens= $^4C_2$ / $^9C_2$ =4 x 3/ 9 x 8= 1/6

After these the pens are not replaced. Therefore there are only 7 pens left.

Probability of drawing 1 black pen from 7 pens= $^3C1$ / $^7C1$ = 3/7

Probability of drawing 2 blue pen and 1 black pen= 1/6 x 3/7 = 1/14

Q5c) Let A be a set of integers, let R be a relation on AxA defined by (a,b) R (c,d) if and only if a+d=b+c.

Prove that R is an equivalent relation.

Solution: **(8)**

Relation R is defined by (a,b) R (c,d) if and only if a+d=b+c.

I) Put a=c and b=d in a+d=b+c

∴ c+d=d+c, which is true

∴ (c,d) R (c,d)

Therefore R is reflexive.

II) Let (a,b) R(c,d)

∴ a+d=b+c

∴ b+c=a+d

∴ c+b=d+a

∴ (c,d)R(a,b)

∴ R is symmetric.

III) Let (a,b)R(c,d) and (c,d)R(e,f)

∴ a+d=b+c …………………………………(1)

And c+f=d+c …………………………………(2)

Adding (1) and (2)

(a+d)+(c+f)=(b+c)+(d+c)

∴ a+f=b+e

∴ (a,b)R(e,f)

Hence R is an equivalence relation.

Q6a) Define reflexive closure and symmetric closure of a relation. Also find reflexive and symmetric closure of R.

A={1,2,3,4}

B={(1,1),(1,2),(1,4),(2,4),(3,1),(3,2),(4,2),(4,3),(4,4)}

Solution: **(4)**

Reflexive closure: A relation R' is the reflexive closure of a relation R if and only if
(1) R' is reflexive,
(2) $R \subseteq R'$, and
(3) for any relation R'', if $R \subseteq R''$ and R'' is reflexive, then $R' \subseteq R''$, that is, R' is the smallest relation that satisfies (1) and (2).

Symmetric closure:A relation R' is the symmetric closure of a relation R if and only if
(1) R' is symmetric,
(2) $R \subseteq R'$, and
(3) for any relation R'', if $R \subseteq R''$, and R'' is symmetric, then $R' \subseteq R''$, that is, R' is the smallest relation that satisfies (1) and (2).

A={1,2,3,4}

B={(1,1),(1,2),(1,4),(2,4),(3,1),(3,2),(4,2),(4,3),(4,4)}

If Δ is euality relation then Reflexive closure R1 = Δ ∪ R ……….(1)

Symmetric closure R2 = R U R^{-1} …………..(2)

Reflexive closure:

Δ={(1,1),(2,2),(3,3),(4,4)}

From (1)

Reflexive closure R1=Δ ∪ R=
{(1,1),(1,2),(1,4),(2,2),(2,4),(3,1),(3,2),(3,3),(4,2),(4,3),(4,4)}

Symmetric closure:

R^{-1}= {(1,1),(2,1),(4,1),(4,2),(1,3),(2,3),(2,4),(3,4)}

R2=R U R^{-1}= {(1,1),(1,2),(1,3),(1,4),(2,1),(2,3),(2,4),(3,1),(3,2),(3,4),(4,1),(4,2),(4,3)}

Q6b) let H= $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Be a parity matrix. Determine the group code B^3 ---> B^6

Solution: **(8)**

Let e_H: B^m --->B^n be encoding function.

If $b=b_1b_2b_3\ldots\ldots b_m$ then

$e_H(b)=b_1b_2b_3\ldots\ldots b_m\ x_1x_2x_3\ldots\ldots x_r$……………(1)

Where r=n-m and

$X_r=b_1.h_{1r}+ b_2.h_{2r}+b_3.h_{3r}...b_m.h_{mr}$……………….(2)

Let B={0,1}

∴B^3={000,001,010,011,100,101,110,111}

Given $B^3 \dashrightarrow B^6$ and H= $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$ $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Here m=3, n=6

$h_{11}=1$, $h_{12}=0$, $h_{13}=0$, $h_{21}=0$, $h_{22}=1$, $h_{23}=1$, $h_{31}=1$, $h_{32}=1$, $h_{33}=1$

∴r=n-m=6-3=3

- For b=000, $b_1=0, b_2=0, b_3=0$

∴From (1), $e_H(000)=b_1b_2b_3x_1x_2x_3$

$=000x_1x_2x_3$

∴From (2), $x_r=b_1.h_{1r}+ b_2.h_{2r}+b_3.h_{3r}$

$=0.h_{1r}+ 0.h_{2r}+0.h_{3r}=0$

∴ $x_1=0$, $x_2=0$, $x_3=o$

∴$e_H(000)=000000$

- For b=001, $b_1=0, b_2=0, b_3=1$

∴From (1), $e_H(001)=b_1b_2b_3x_1x_2x_3$

$=001x_1x_2x_3$

∴From (2), $x_r=b_1.h_{1r}+ b_2.h_{2r}+b_3.h_{3r}$

$=0.h_{1r}+ 0.h_{2r}+1.h_{3r}=h_{3r}$

$\therefore$ $x_1=h_{31}=1$,$x_2=h_{32}=1$, $x_3=h_{33}=1$

$\therefore e_H(001)=000111$

◆ For b=010, $b_1=0,b_2=1,b_3=0$

$\therefore$From (1), $e_H(010)=b_1b_2b_3x_1x_2x_3$

$=010x_1x_2x_3$

$\therefore$From (2), $x_r=b_1.h_{1r}+ b_2.h_{2r}+b_3.h_{3r}$

$=0.h_{1r}+ 1.h_{2r}+0.h_{3r}=h_{2r}$

$\therefore$ $x_1=h_{21}=0$,$x_2=h_{22}=1$, $x_3=h_{23}=1$

$\therefore e_H(010)=010011$

◆ For b=011, $b_1=0,b_2=1,b_3=1$

$\therefore$From (1), $e_H(011)=b_1b_2b_3x_1x_2x_3$

$=011x_1x_2x_3$

$\therefore$From (2), $x_r=b_1.h_{1r}+ b_2.h_{2r}+b_3.h_{3r}$

$=0.h_{1r}+ 1.h_{2r}+1.h_{3r}=h_{2r}+h_{3r}$

$\therefore$ $x_1=h_{21}+h_{31}=0+1=1$,$x_2=h_{22}+h_{32}=1+1=0$, $x_3=h_{32}+h_{33}=1+1=0$

$\therefore e_H(011)=011100$

◆ For b=100, $b_1=1,b_2=1,b_3=0$

$\therefore$From (1), $e_H(100)=b_1b_2b_3x_1x_2x_3$

$=100x_1x_2x_3$

$\therefore$From (2), $x_r=b_1.h_{1r}+ b_2.h_{2r}+b_3.h_{3r}$

$=1.h_{1r}+ 0.h_{2r}+0.h_{3r}=h_{1r}$

$\therefore x_1=h_{11}=1 , x_2=h_{12}=0, x_3=h_{13}=0$

$\therefore e_H(100)=100100$

- For b=101, $b_1=1,b_2=0,b_3=1$

$\therefore$From (1), $e_H(101)=b_1b_2b_3x_1x_2x_3$

$=101x_1x_2x_3$

$\therefore$From (2), $x_r=b_1.h_{1r}+ b_2.h_{2r}+b_3.h_{3r}$

$=1.h_{1r}+ 0.h_{2r}+1.h_{3r}=h_{1r}+h_{3r}$

$\therefore x_1=h_{11}+h_{31}=1+1=0 , x_2=h_{12}+h_{32}=0+1=1, x_3=h_{13}+h_{33}=0+1=1$

$\therefore e_H(101)=101011$

- For b=110, $b_1=1,b_2=1,b_3=0$

$\therefore$From (1), $e_H(110)=b_1b_2b_3x_1x_2x_3$

$=110x_1x_2x_3$

$\therefore$From (2), $x_r=b_1.h_{1r}+ b_2.h_{2r}+b_3.h_{3r}$

$=1.h_{1r}+ 1.h_{2r}+0.h_{3r}=h_{1r}+h_{2r}$

$\therefore x_1=h_{11}+h_{21}=1+0=1 , x_2=h_{12}+h_{22}=0+1=1, x_3=h_{13}+h_{23}=0+1=1$

$\therefore e_H(110)=110111$

- For b=111, $b_1=1,b_2=1,b_3=1$

$\therefore$From (1), $e_H(111)=b_1b_2b_3x_1x_2x_3$

$=111x_1x_2x_3$

$\therefore$From (2), $x_r=b_1.h_{1r}+ b_2.h_{2r}+b_3.h_{3r}$

$=1.h_{1r}+ 1.h_{2r}+1.h_{3r}=h_{1r}+h_{2r}+h_{3r}$

$\therefore x_1=h_{11}+h_{21}+h_{31}=1+0+1=0$,

$x_2=h_{12}+h_{22}+h_{32}=0+1+1=0$,

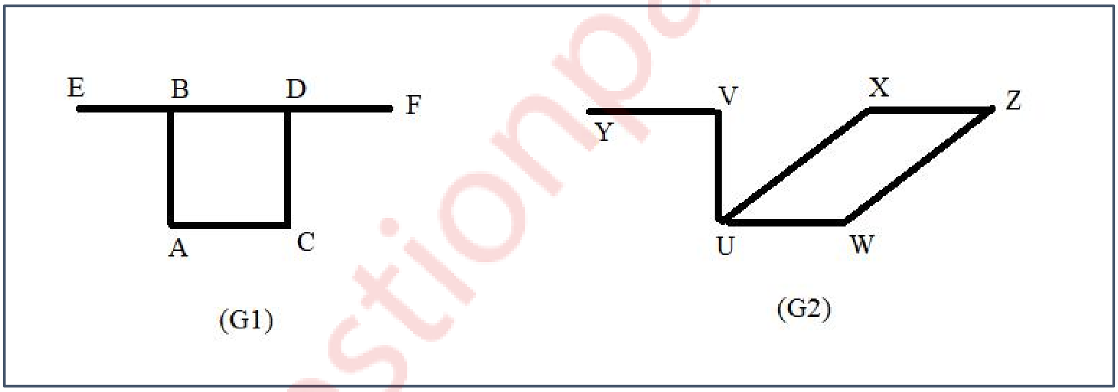
$x_3=h_{13}+h_{23}+h_{33}=0+1+1=0$

$\therefore e_H(111)=111000$

Hence the(3,6) encoding function e_H:B^3 ---> B^6 is defined as:

e(000)=000000,e(001)=001111,e(010)=010011,e(011)=010011,e(100)=100100,e(101)=101011,e(110)=110111,e(111)=111000.

Q6c) Determine if following graphs G1 and G2 are isomorphic or not. (8)



Solution:

Two graphs G(V,E) and H(V',E') isomorphic if

1) There is one to one correspondence f from V to V' such that f(V1)=V1' and f(V2)=V2' for every (v1, v2)Є V and (v1', v2')Є V'
2) G and H should have equal no. Of edges.
3) G and H should have equal no. Of vertices.
4) G and H should have same degree of vertices
5) Adjacency property is observed in each vertex.

From the above two graphs:

Graph G1		
Number of vertices	6	
Number of Edges	6	
Vertex	**Degree of vertex**	**Adjacent vertices**
A	1	B(3)
B	3	A(1), C(3), E(2)
C	3	B(3), D(1), F(2)
D	1	C(3)
E	2	B(3), F(2)
F	2	E(3),C(2)

Graph G2		
Number of vertices	6	
Number of Edges	6	
Vertex	**Degree of vertex**	**Adjacent vertices**
Y	1	V(2)
V	2	Y(1), U(3)
X	2	Z(2), U(3)
Z	2	X(2),W(2)
U	3	V(2), X(2),W(2)
W	2	U(3),Z(2)

We observe that,

There are equal no of edges and vertices for both graph.

Graph G1 has 2 vertices with degree 1, two vertices with degree 2 and two vertices with degree 2.

Graph G2 has one vertices with degree 1, four vertices with degree 2 and one vertex with degree 3.

Therefore, the degree in two graphs are not equal.

Hence the two graphs are not isomorphic.

DISCRETE STRUCTURE

COMPUTER ENGINEERING

MAY 2018

Q1.a) Prove by induction that the sum of the cubes of three consecutive numbers is divisible by 9. [5]

Solution :- Let the consecutive numbers be n, n+1, n+2

P(n): $n^3 + (n+1)^3 + (n+2)^3 = 9k$

LHS : $1^3 + 2^3 + 3^3 \Rightarrow 1 + 8 + 27 = 36$

36 is divisible by 9

Let us assume P(k) is true;

LHS : $k^3 + (k+1)^3 + (k+2)^3 = 9m$

$\Rightarrow (k+1)^3 + (k+2)^3 = 9m - k^3$

Let us prove P(k+1) is true;

LHS : $(k+1)^3 + (k+2)^3 + (k+3)^3$

$\Rightarrow 9m - k^3 + (k+3)^3$

$\Rightarrow 9m - k^3 + k^3 + 9k^2 + 27k + 27$

$\Rightarrow 9m + 9k^2 + 27k + 27$

$\Rightarrow 9(m + k^2 + 3k + 3)$

Which implies P(k+1) is true

Therefore, P(k) is true

Therefore, P(n) is true.

Hence for $n \in N$, the result is true.

Q1.b) Find the generating function for the finite sequences.

i)2,2,2,2,2,2 ii)1,1,1,1,1,1 [5]

Solution:

1. Multiplying the sequence successively by $x^0, x^1, x^2, x^3, \ldots$

$2x^0 + 2x^1 + 2x^2 + 2x^3 + \ldots$

$2(x^0 + x^1 + x^2 + x^3 + \ldots)$

$2(1-x)^{-1}$

Ans : $\frac{2}{1-x}$

2. Multiplying the sequence successively by $x^0, x^1, x^2, x^3, \ldots$

$x^0 + x^1 + x^2 + x^3 + \ldots$

$(x^0 + x^1 + x^2 + x^3 + \ldots)$

$(1-x)^{-1}$

Ans : $\frac{1}{1-x}$

Q1.c) A box contains 6 white balls and red balls. In how many ways 4 balls can be drawn from the box if, i) they are to be of any color ii)all the balls to be of the same color. [5]

Solution :

1. There are 11 balls and 4 are to be drawn.

This can be done in $11C_4 = 330$

2. All the balls of same color:

4 white balls can be drawn from $6C_4$

4 red balls can be drawn from $5C_4$

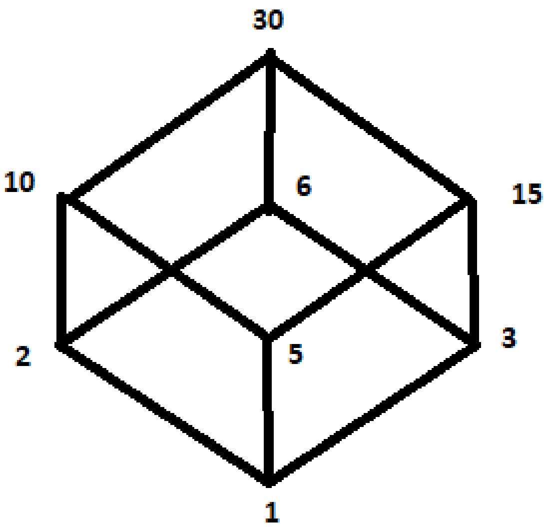
Number of ways = $6C_4 . 5C_4$ = 15 x 5 = 75

Q1.d) Find the complement of each element in D_{30} **[5]**

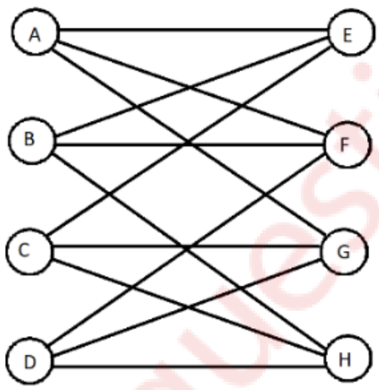
Sol : $D_{30} = \{1,2,3,5,6,10,15,30\}$

1	˅	30
2	˅	30
3	˅	30
5	˅	30
6	˅	30
10	˅	30
15	˅	30
30	˅	30

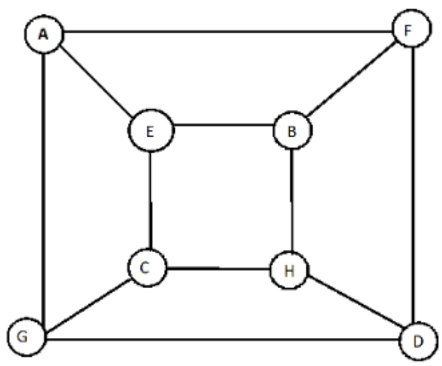
1	^	1
2	^	1
3	^	1
5	^	1
6	^	1
10	^	1
15	^	1
30	^	1



Q2.a) Define Isomorphism of graphs. Find if the following two graphs are isomorphic. If yes, find the one-to-one correspondence between the vertices. [8]



G1



G2

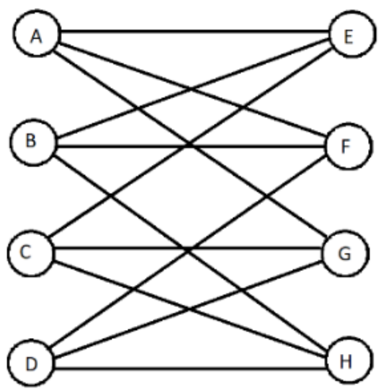
Solution :

If two graphs are isomorphic then:

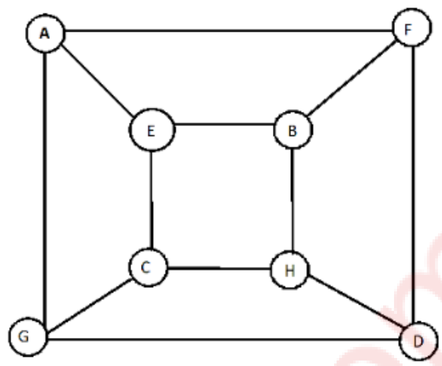
i) They must have same number of vertices.

ii) They must have same number of edges.

iii) They must have same degree of vertices.



G1



G2

n(G1)=8

no. of edges=12

degree of each vertex=3

n(G2)=8

no. of edges=12

degree of each vertex=3

Ans: The graphs are isomorphic.

Q2.b) In a certain college 4% of the boys and 1% of the girls are taller than 1.8 mts. Furthermore 60% of the students are girls. I a student at random is taller than 1.8 mts, what is the probability that the student was a boy? Justify your answer. [8]

Solution:

For the convenience suppose there are 1000 students in the college.

Therefore,

	Taller than 1.8	Less than 1.8	Total
Boys	16	384	400
Girls	6	594	600
Total	22	978	1000

Since the student selected at random is found to be taller than 1.8, the student is one of 22.

But one of 22 students, 16 are boys

Therefore, the required probability = $\frac{16}{22} = \frac{8}{11}$

Ans : 8/11

Q2.c) Prove $\sim (p \vee (\sim p \wedge q))$ and $\sim p \wedge \sim q$ are logically equivalent by developing a series of logical equivalences. [4]

Solution :

By Distributive law,

=> $\sim ((\sim p \vee p) \wedge (p \vee q))$

=> $\sim (T \wedge (p \vee q))$ $[(\sim p \vee p) = T]$

=> $\sim ((p \vee q))$

=> $\sim p \wedge \sim q$ By Demorgan's

Q3.a) Prove that set G={1,2,3,4,5,6} is a finite abelian group of order 6 with respect to multiplication modulo 7. [8]

Solution:

X_7	**1**	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

From the table, X_7 is associative

Eg : 2 X_7 (3 X_7 5) = 2 X_7 1=2

(2 X_7 3) X_7 5) = 6 X_7 5=2

$a X_7 e = a$

Here e=1, identity element = 1

$a X_7 a^{-1} = e$

Every element has a multiplicative inverse.

Also, $a X_7 e = b X_7 a$

$4 X_7 5 = 6$

$5 X_7 4 = 6$

Ans : Therefore, G is abelian group.

Q3.b)LetA={1,2,3,4,5}, let R={{1,1},{1,2},{2,1},{2,2},{3,3},{3,4},{4,3},{4,4},{5,5}} and S={{1,1},{2,2},{3,3},{4,4},{4,5},{5,4},{5,5}} be the relations on A. Find the smallest equivalence containing relation R and S. [8]

Solution.

$$M_R: \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$M_S: \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

$$M_{RUS} = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

Finding W5 by Warshall's Algorithm,

C1: 1 is at 1,2

R1: 1 is at 1,2

Put 1 in (1,1), (1,2), (2,1), (2,2)

$$W1 = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

C2: 1 is at 1,2

R2: 1 is at 1,2

Put 1 in (1,1), (1,2), (2,1), (2,2)

$$W2 = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

C3: 1 is at 3,4

R3: 1 is at 3,4

Put 1 in (3,3), (3,4), (4,3), (4,4)

$$\begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{vmatrix}$$

$$W3 = \begin{vmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

C4: 1 is at 4,5,6

R4: 1 is at 4,5,6

Put 1 in (4,4), (4,5), (4,6), (5,4), (5,5), (5,6), (6,4), (6,5), (6,6)

$$W4 = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

C5: 1 is at 5,6

R5: 1 is at 5,6

Put 1 in (5,5), (5,6), (6,5), (6,6)

$$W5 = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

Therefore, transitive closure = {(1,1),(1,2),(2,1),(3,3),(3,4),(4,3),(4,4),(4,5),(5,4),(5,5)}

Q3.c) Test whether the following function is one-to-one, onto or both.

f:Z→Z, f(x) = x^2 + x + 1 **[8]**

Sol : The set is Z

To test whether function is injective,

$f(x_1) = f(x_2)$

$\Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow (x_2 - x_1)(x_1 + x_2 + 1) = 0$

Therefore, $x_2 - x_1 = 0;$ $\quad$ $x_1 + x_2 + 1 = 0$

$x_2 = x_1\ ;$ $\quad$ $x_2 = -1 - x_1$

Therefore, f is not injective.

Now, $y = x^2 + x + 1$

If y=0; $x_1 + x_2 + 1 = 0$

$$x = \frac{-1 \pm \sqrt{1-4}}{2} \Rightarrow \text{ x is imaginary, ie, } \notin Z$$

Therefore, f is not surjective.

Ans: The function f is not one-to-one or onto

Q4.a) Show that the (2,5) encoding function e:$B^2 \rightarrow B^5$ defined by

e(00)=00000 e(01)=01110

e(10)=10101 e(11)=11011 is a group code.

How many errors will it detect and correct? [8]

Sol:

$\oplus$	**00000**	**01110**	**10101**	**11011**
00000	00000	01110	10101	11011
01110	01110	00000	11011	10101
10101	10101	11011	00000	01110
11011	11011	10101	01110	00000

From diagonal elements, 00000 is identity element.

$x \oplus y \in N$ for $x,y \in N$

Every element has its inverse.

Therefore, e is a groupcode.

00000,01110=δ(w,x)=3

00000,10101=δ(w,y)=3

00000,11011=δ(w,z)=4

01110,10101=δ(x,y)=4

01110,11011=δ(x,z)=3

10101,11011=δ(y,z)=3

Therefore, the minimum distance = 3

k+1=3,

k=2

Ans: It can detect 2 or less errors.

Q4.b) Let H= $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ **be a parity check matrix. Determine the group code eH:**$B^3 \rightarrow B^6$

[8]

Sol:

B^3 = {000,001,010,011,100,101,110,111}

$x_1 = b_1h_{11} + b_2h_{21} + b_3h_{31}$

$x_2 = b_1h_{12} + b_2h_{22} + b_3h_{32}$

$x_3 = b_1h_{13} + b_2h_{23} + b_3h_{33}$

But $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Substituting the values;

$e(000) = 000\, x_1\, x_2\, x_3$

$x_1 = 0.1 + 0.0 + 0.1 = 0$

$x_2 = 0.0 + 0.1 + 0.1 = 0$

$x_3 = 0.0 + 0.1 + 0.1 = 0$

$e(001) = 001\, x_1\, x_2\, x_3$

$x_1 = 0.1 + 0.0 + 1.1 = 1$

$x_2 = 0.0 + 0.1 + 1.1 = 1$

$x_3 = 0.0 + 0.1 + 1.1 = 1$

$e(010) = 010\, x_1\, x_2\, x_3$

$x_1 = 0.1 + 1.0 + 0.1 = 0$

$x_2 = 0.0 + 1.1 + 0.1 = 1$

$x_3 = 0.0 + 1.1 + 0.1 = 1$

$e(011) = 011\, x_1\, x_2\, x_3$

$x_1 = 0.1 + 1.0 + 1.1 = 0$

$x_2 = 0.0 + 1.1 + 1.1 = 0$

$x_3 = 0.0 + 1.1 + 1.1 = 0$

$e(100) = 100\ x_1\ x_2\ x_3$

$x_1 = 1.1 + 0.0 + 0.1 = 1$

$x_2 = 1.0 + 0.1 + 0.1 = 0$

$x_3 = 1.0 + 0.1 + 0.1 = 0$

$e(101) = 101\ x_1\ x_2\ x_3$

$x_1 = 1.1 + 0.0 + 1.1 = 0$

$x_2 = 1.0 + 0.1 + 1.1 = 1$

$x_3 = 1.0 + 0.1 + 1.1 = 1$

$e(110) = 110\ x_1\ x_2\ x_3$

$x_1 = 1.1 + 1.0 + 0.1 = 1$

$x_2 = 1.0 + 1.1 + 0.1 = 1$

$x_3 = 1.0 + 1.1 + 0.1 = 1$

$e(111) = 111\ x_1\ x_2\ x_3$

$x_1 = 1.1 + 1.0 + 1.1 = 0$

$x_2 = 1.0 + 1.1 + 1.1 = 0$

$x_3 = 1.0 + 1.1 + 1.1 = 0$

e(000) = 000000

e(001) = 001111

e(010) = 010011

e(011) = 011100

e(100) = 100100

e(101) = 101011

e(110) = 110111

e(111) = 111000

Q4.c) How many friends must you have to guarantee that at least five of them will have birthdays in the same month? [4]

Sol : Since there are twelve months considering even distribution, if there are 48 friends, atleast 4 will have birthday in the same month.

Hence, if we have 49 friends, then five of them will have birthday in the same month.

Or by extended pigeonhole principle,

$\left[\frac{n-1}{12}\right] + 1 = 5$

$\Rightarrow \left[\frac{n-1}{12}\right] = 4$

$\Rightarrow n - 1 = 48$

Therefore, n=49.

Ans: Total friends needed so that at least five of them will have birthdays in the same month = 49

Q5) a] Let G be a set of rational numbers other than 1. Let * be an operation on G defined by a * b = a + b – ab for all a, b ∈ G. Prove that (G, *) is a group (8)

Solution:-

Let a, b ∈ G

Assume, $a + b - ab = 1$

$a - b + b - ab = 0$

$(a - 1)(-b + 1) = 0$

a = 1 and b = 1; since a, b ∈ G the set of rational numbers

$a * (b * c) = a * (b + c - bc) = a + (b + c - bc) - a(b + c + bc)$

$= a + b + c - bc - ab - ac - abc$

G is associative

$a * 0 = 0$; $0 \in G$ is identity

$a * a^{-1} = 0$; a^{-1} exists ∀ (x, y) ∈ R

Q5] b) solve $a_r - 7a_r + 10a_{r-2} = 6 + 8r$ given $a_0 = 1$, $a_1 = 1$ [4mk]

Solution:-

$a_r - 7a_r + 10a_{r-2} = 0$

$r^2 - 7r + 10 = 0$

$(r-2)(r-5) = 0 \quad r = 2;5$

Roots are real rational and distinct

Let the solution be

$a_n^{(n)} = A2^n + B5$

F(r) is linear ; we assume particular solution as ar+b

$(an+b) - 7[a(n-1)+b] + 10[a(n-2)+b] - 6 - 8n = 0$

$(4a-8)n + (-13a+4b-6) = 0$

$4a - 8 = 0$

$a = 2$

$-26 + 4b - 6 = 0$

$b = 8$

Solution:- $a_n = 2(2)^n + 3(5)^n + 2n + 8$

Q5] c) Let A ={a,b,c,d,e,f,g,h}. Consider the following subsets of A

A1 = {a, b, c, d} A2 = {a, c, e, g, h} A3 = {a, c, e, g}

A4 = {b, d} A5={f,h}

Determine whether following is partition of A or not. Justify your answer.

1. {A1,A2} 2. {A3, A4, A5} (8)

Solution:-

For partition $A_1 \cup A_2 \cup \ldots\ldots A_n = A$

And $A_i \cap A_j = \emptyset;\ i \neq j$

Consider $A_1 = \{a,b,c,d\}$ and $A_2 = \{a,c,e,g,h\}$

$A_1 \cup A_2 = \{a,b,c,d\} \cup \{a,c,e,g,h\} = \{a,b,c,d,g,h,e\} \neq A$

$A_1 \cap A_2 = \{a,c\} \neq 0$

$\{A_1,A_2\}$ is not a partition

Consider $A_3, A_4, A_5,$

$A_3 = \{a,c,e,g\};\ A_4 = \{b,d\};\ A_5 = \{f,h\}$

$A_3 \cup A_4 \cup A_5 = \{a,b,c,d,e,f,g,h\} = A$

$A_3 \cap A_4 = \emptyset$

$A_4 \cap A_5 = \emptyset$

$A_3 \cap A_5 = \emptyset$

A_3,A_4,A_5 is a partition.

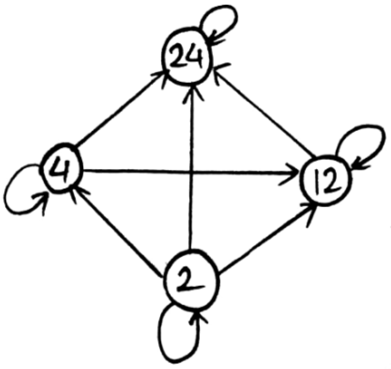
Q6] a) Draw the Hasse Diagram of the following sets under the partial order relation divides and indicate which are chains. Justify your answers. (8)

1. **A = {2,4,12,24}**
2. **A = {1,3,5,15,30}**

Solution:-

The given partial order relation is divides

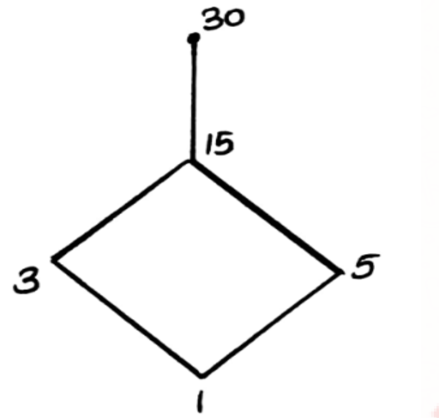
1. A = {2,4,12,24}

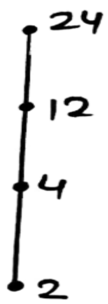


2.

The poset is a chain

3. B = {1,3,5,15,30}





The poset is not a chain

Q6] b) Let the functions f, g and h defined as follows: (8)

f: R -> R, f(x) = 2x + 3

g: R->R, g(x) =3x + 4

h: R -> R,h(x) = 4x

find gof, fog, foh and gofoh

Solution:-

1. gof = g(f(x))

   = g(2x+3)

   = 3(2x+3)+4

   = 6x+9+4

   = 6x+13

2. fog = f(g(x))

   = f(3x+4)

   = 2(3x+4)+3

   = 6x+8+3

   = 6x+11

3. foh = f(h(x))

   = f(4x)

   = 2(4x)+3

   = 8x+3

4. gofoh = g(f(h(x)))

   = g(f(4x))

   = g(2(4x)+3)

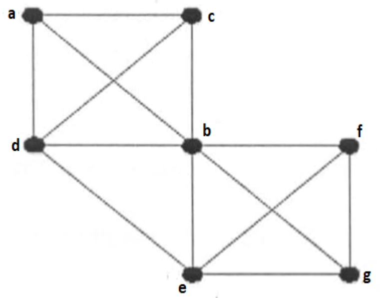
   = g(8x+3)

   = 24x+9+4

= 24x+13

Q6] c) Determine Euler Cycle and path in graph shown below (4)

Solution:-



No Eulerian path exists as 4 vertices are of odd degree i.e a, c, g, f are of odd degree = 3

The graph is not Eulerian as not every vertex has even degree.
